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STRESS AND THERMAL ANALYSIS OF THE THROAT SECTIONS 50-INCH MACH 10 - 12 TUNNEL (C)

By

R. Sherman and J. P. Cook
von Kármán Gas Dynamics Facility
ARO, Inc.

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ARNOLD ENGINEERING DEVELOPMENT CENTER

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

NOTICES

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ARO, Inc.
a subsidiary of Sverdrup and Parcel, Inc.

February 1963

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ABSTRACT

This report describes the design analysis of the throat section for a large (50-inch-diameter test section), continuous flow, axisymmetric wind tunnel which is currently in operation at the AEDC. The maximum stagnation conditions are 2000 psia and 1450°F.

Several problems which must be considered before design are cited, and possible means of solution are discussed. The "as-built" article, which in essence is simply a water-cooled liner housed in a pressure shell, is described, and a method, which allows reasonable selections of geometry and cooling requirements, outlined. This method is extended to enable forecast of temperature distribution and resultant stress along the throat section and is applied, not only to the Mach 10 configuration, but also to a proposed interchangeable counterpart which would permit aerodynamic testing at Mach 12. Maximum stagnation conditions therein would be 2400 psia and 1940°F.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

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NOMENCLATURE

A	Area, $in.^2$
C_f	Skin friction coefficient
cp	Specific heat at constant pressure, Btu/lb °F
D	Hydraulic diameter, ft
d	Diameter, in.
E	Young's modulus, $lb/in.^2$
f	Friction coefficient
G	Mass rate of flow, $lb/hr\,ft^2$
g	Acceleration due to gravity, ft/sec^2
h	Heat transfer coefficient, $Btu/sec\,ft^2\,°F$
K	Ratio of $r_2:r_1$
$\bar{K}$	Conductivity, $Btu/hr\,ft^2\,°F/ft$
La	Axial load, lb
ℓ	Length, in.
M	Mach number
Pr	Prandtl number, $cp\nu/\bar{K}$
p	Pressure, $lb/in.^2$
q	Rate of heat flow, Btu/sec
R	Gas constant, ft/°R
Re	Reynolds number, $(\rho V/\nu)$ x a characteristic length
R^*	Longitudinal radius of curvature at the throat, in.
r	Radius, in.
r_f	Recovery factor
r_h	Hydraulic radius, ft
St	Stanton number, $h/cpV\rho$
T	Temperature, °R; °F
t	Thickness, in.
V	Velocity, ft/sec
v	Specific volume, ft^3/lb
X	Water flow rate, gpm

x	A distance along the nozzle, ft
α	Coefficient of thermal expansion, in./in. °F
γ	Ratio of specific heats
$\bar{\gamma}$	Coefficient of thermal stress, lb/in.^{2} °F
Δ	An increment
ν	Viscosity, lb/hr ft
$\bar{\nu}$	Poisson's ratio
ρ	Density, lb/ft^3
σ	Stress, lb/in.^{2}

SUBSCRIPTS

a	At the air-side wall
b	Denotes bulk temperature of the water
f	Due to friction
i	Initial
L	Due to axial load
ℓ	Longitudinal
m	Mean
o	At stagnation conditions
p	Due to pressure
r	Radial; also, at a radius; also, recovery temperature
s	Free-stream conditions
T	Due to temperature
t	Tangential
w	At water-side wall; also, working stress
x	Versus station
1, 2, · · ·	Defined by sketch, in Section 4.1

SUPERSCRIPTS

*	At the throat
′	At Eckert's reference temperature

1.0 INTRODUCTION

A continuous flow, Mach 10, axisymmetric wind tunnel, with 50-inch-diameter test section, is currently in operation at the von Kármán Gas Dynamics Facility (VKF), Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). Maximum stagnation conditions are 2000 psia and 1450°F.

This report is concerned with the design analysis of the present throat section and of a proposed interchangeable counterpart for use at a Mach 12. Sectional views of each are shown in Figs. 1 and 2. Other components of the tunnel are described in detail in Ref. 1.

2.0 DESIGN REQUIREMENTS AND CONSIDERATIONS

Several problems are inherent in the design of the throat section of a hypersonic wind tunnel, particularly one of large size in which temperatures approach 2000°F and operation is to be continuous. First, because of aerodynamic considerations, the contour must remain stable, smooth, and continuous, at least downstream of the location where the Mach number is 0.1. Secondly, the material selected must be capable of sustaining the stresses induced by the pressure and thermal gradients and of withstanding the corrosive and errosive actions of the airflow. In addition, ease of fabrication, assembly and maintenance, and the requirements of personnel safety must be met. At least two avenues of approach are readily apparent; these are discussed herein.

Consider first an uncooled liner, perhaps made from a ceramic, cermet, high temperature metal, or metal alloy, cast or machined to the required aerodynamic contour and enclosed in a steel pressure shell. The material choice, if fabricated from ceramic or cermet, would depend primarily on an ability to withstand severe thermal shock. Strength, though important, becomes a secondary consideration because of the feasibility of applying static pressure, somewhat above the maximum stagnation, on the outside of the liner. The resultant pressure stresses are thus transformed from tension to compression, an ideal situation from the standpoint of most ceramics or cermets. Temperature expansion forces may be minimized by the choice of a material with a low

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coefficient of thermal expansion and, if necessary, by designing the liner in two sections, opposing ends attached to the pressure shell and near ends telescoping one into the other. Packing the cavity between liner and shell with insulation and allowing the external pressure (assumed to enter at relatively low temperature) to bleed in controlled quantity through the annulus around the telescoping joint would ensure a reasonable shell temperature and prevent the formation of hot spots caused by thermal circulation. A similar configuration might be made from a high temperature metal or metal alloy. The choice of material, in this case, would be based on resistance to scaling in air at high temperature and pressure, low thermal coefficient of expansion, and at least a modest creep strength.

As a second possibility, consider the use of a metal, water-cooled liner enclosed, as before, in a steel pressure shell which serves to contain the water and, in addition, provides back-up structure in the event of liner failure. Such a scheme obviates the former requirement of thermal shock resistance necessary for a ceramic liner and minimizes the thermal expansion problems inherent in the uncooled metal version. A liner made in two telescoping sections would still be mandatory because of axial growth; however, again as before, cool high pressure air in controlled quantity might be bled through the joint annulus and thereby effect a degree of boundary-layer cooling. In addition, this circumvents considerable difficulty in providing a positive seal between two surfaces which are subjected to bi-directional movement. For such a configuration, the liner must be capable of withstanding the combined effects of water and air pressure and, in addition, effectively transmit heat to the water. Therefore, the material choice depends on high thermal conductivity and yield strength and on low modulus and coefficient of thermal expansion, all at the resultant operating temperature of the metal. Arranged on the following page in preferencial order, for an operating temperature of 650°F, are some of the obvious possibilities within the scope of commercially available materials.

Material	σ_{allow}	$\bar{K}$	α	E	$\sigma\bar{K}/\alpha E$
	psi x 10^{-3}	$\frac{Btu/ft}{hr\ ft^2\ °F}$	$\frac{in.\ x\ 10^6}{in.\ °F}$	psi x 10^{-6}	$\frac{Btu\ x\ 10^{-3}}{hr\ ft}$
Berylco 10	95	165	10.3	17.5	87.0
Berylco 25	150	82	10	19	64.7
Zr-Cu	34.8	205	11.2	17	47.0
Molybdenum	>35	74	2.6	<42	>23.7
Tungsten	47.8	71.7	2.6	57	23.1
Al-Bronze (92-8)	7.5	>42	10	<15	>21.0
Silver	< 8	215	11.1	7.8	<19.9
Cu-Ni (70-30)	9.8	25	9	16.4	16.6
A-L-25 Ni (Maraging)	250	~12	<8	<23	~16.3
Hi Carbon Steel	100	24.5	7.3	25.6	13.1
Cr-Cu	<13	>187	10	<19	~12.8
Ferritic Stainless Steel	109	13	6.2	25.4	9.0
Low Alloy Steel (4130)	100	16.5	7.3	27.0	8.4
17-4PH Stainless Steel	92	12	6.4	26	6.6
Hi Alloy Steel (J-1300)	120	9.7	8.7	25.4	5.3
Pure Copper	< 2.6	217	9.8	13.9	< 4.1
Ti (4Al-3 Mo-IV)	23	9	5	14.5	2.9
Austenitic Stainless Steel	45.5	11.5	9.1	24.3	2.4
Electrolytic Nickel	18.1	28.5	9.3	27.2	2.0
Monel	21.5	17.3	9.7	24	1.6

3.0 CHOICE AND DESCRIPTION OF DESIGN

Juxtaposition of the design possibilities outlined previously leads to the choice of a water-cooled throat section. The feasibility of this selection is justified in later sections; however it should be noted that Mach 12, for the size of the tunnel under consideration, is near the upper limit of such a configuration because of the high heat transfer rates involved. A brief description of the geometry that was finally chosen follows.

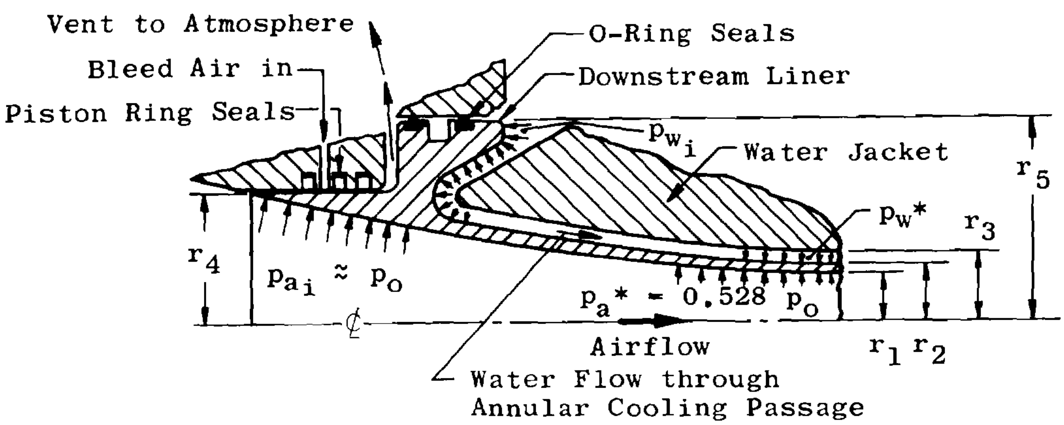
As shown in the table in Section 2.0, Berylco-10 has the highest ratio of $\sigma\bar{K}/\alpha E$ and is judged, therefore, to be the material most suitable for the liner, which is made in two sections, machined to form the

aerodynamic contour, and enclosed in a thick walled pressure shell (see Figs. 1 and 2). Water supplied from a high pressure source is forced through longitudinal grooves cut in the upstream section and, downstream, through an annular passage between the liner and a surrounding jacket. A telescoping joint, the entrance of which is located in the region of low sub-sonic flow (i.e., the Mach number ≤ 0.1), is provided. Three piston rings are contained within the joint, and air, coming from the tunnel supply source but bypassing a propane-fired heater in the flow circuit, is introduced between the upstream pair of rings. This air, arriving at heat of compression temperature and a pressure somewhat above stagnation, then spills in two directions, a portion into the tunnel and the balance past the third ring and out a vent to atmosphere. A flange, integral with the downstream liner, retains the water seals on that section and is sized to counterbalance, by means of water pressure, the axial load of the air. For the Mach 10 throat section now in operation, raw water is supplied at the rate of 325 gpm with an inlet pressure of 400 psig and dumped to drain; for Mach 12 operation, 750 gpm of distilled water at 1150 psig would be required and, for reasons of economy, would be contained within a closed loop.[†]

4.0 GENERAL ANALYSIS

4.1 BASIC GEOMETRY AND EQUATIONS

Consider the free body, which shows a portion of the downstream liner section under the action of pressure forces from tunnel air and cooling water.



[†]Separate cooling circuits are provided for the up and downstream liners; 50 gpm are used (or will be used) for the former at either Mach number.

Basic stress equations, written for a differential element at the throat are:

When $r = r_1$:

$$\sigma_t = 0.5283\, p_o \left\{\frac{K^2+1}{K^2-1}\right\} - \frac{2\, p_w^*}{K^2-1} - \bar{\gamma}_a\,(T_a - T_w) \qquad \text{Eq. (1)}$$

$$\sigma_\ell = \frac{p_w\,(r_3^2 - r_2^2)}{r_1^2\,(K^2-1)} - \frac{p_o\,(r_4^2 - r_1^2)}{r_1^2\,(K^2-1)} - \bar{\gamma}_a\,(T_a - T_w) \qquad \text{Eq. (2)}$$

$$\sigma_r = 0.5283\, p_o \qquad \text{Eq. (3)}$$

When $r = r_2$:

$$\sigma_t = \frac{1.057\, p_o}{(K^2-1)} - p_w^* \left\{\frac{K^2+1}{K^2-1}\right\} + \bar{\gamma}_w(T_a - T_w) \qquad \text{Eq. (4)}$$

$$\sigma_\ell = \frac{p_w\,(r_3^2 - r_2^2)}{r_1^2\,(K^2-1)} - \frac{p_o\,(r_4^2 - r_1^2)}{r_1^2\,(K^2-1)} + \bar{\gamma}_w\,(T_a - T_w) \qquad \text{Eq. (5)}$$

$$\sigma_r = -\, p_w^* \qquad \text{Eq. (6)}$$

If steady-state heat transfer is considered, the thermal stress coefficients, $\bar{\gamma}_a$ and $\bar{\gamma}_w$, become:

$$\bar{\gamma}_a = \frac{E\,a}{2(1-\bar{\nu})\,ln\,K}\left\{1 - \frac{2K^2\,ln\,K}{K^2-1}\right\},\ \text{psi/deg} \qquad \text{Eq. (7)}$$

$$\bar{\gamma}_w = \frac{E\,a}{2(1-\bar{\nu})\,ln\,K}\left\{1 - \frac{2\,ln\,K}{K^2-1}\right\},\ \text{psi/deg} \qquad \text{Eq. (8)}$$

The preceding may be substituted into the generally accepted theory of failure for ductile materials under the action of hydrostatic loads, viz., that of shear-distortion which is written:

$$(\sigma_t - \sigma_\ell)^2 + (\sigma_t - \sigma_r)^2 + (\sigma_\ell - \sigma_r)^2 = 2\sigma^2_{total} \qquad \text{Eq. (9)}$$

Now, consider a thermal balance which must satisfy the identity:

$$h_a\,\pi d_1 \setminus \ell(T_r - T_a) \equiv \frac{\bar{K}}{t}\,\pi d_m \setminus \ell(T_a - T_w) \equiv h_w\,\pi d_2 \setminus \ell(T_w - T_b) \qquad \text{Eq. (10)}$$

Variables, which may be put in equation form for use in the above, are:

$$T_r = T_s + r_f(T_o - T_s) \qquad \text{Eq. (11)}$$

$$\bar{K} = \text{some linear function of temperature} \qquad \text{Eq. (12)}$$

$$h_w = \frac{0.023\, cp^{1/3}}{\nu_w^{\,0.14}}\left\{\frac{\bar{K}^{2/3}}{\nu^{1/3}}\right\}\frac{G^{0.8}}{D^{0.2}} \quad \text{(The Colhurn equation, Ref. 2)} \qquad \text{Eq. (13)}$$

Finally, for calculation of pressure requirements, the Bernoulli equation reduces to:

$$\Delta p_{total} = \left\{\frac{V_1^2 - V_2^2}{2g}\right\}\rho + \Delta p_{friction} \qquad \text{Eq. (14)}$$

where

$$\Delta p_{friction} = \frac{f G^2 v \ell}{2 g r_h} \qquad \text{Eq. (15)}$$

and

$$f = 0.00140 + \frac{0.125}{Re^{0.32}} \qquad \text{(Ref. 2)} \qquad \text{Eq. (16)}$$

4.2 OPTIMUM WALL THICKNESS AT THE THROAT

Because of the small contribution of radial to the total combined stress, the system may be considered biaxial. Dropping σ_r, then, from the theory of failure equation and noting that the material is used most efficiently when the total combined stress on an inside element is equal in magnitude to one on the outside, and with appropriate subscripts:

$$\begin{aligned}&\left\{\sigma_{t_p} + \sigma_{t_T}\right\}^2_{r=r_1} - \left[\left\{\sigma_{t_p} + \sigma_{t_T}\right\}\left\{\sigma_{\ell_L} + \sigma_{\ell_T}\right\}\right]_{r=r_1} + \left\{\sigma_{\ell_L} + \sigma_{\ell_T}\right\}^2_{r=r_1} \\ &= \left\{\sigma_{t_p} + \sigma_{t_T}\right\}^2_{r=r_2} - \left[\left\{\sigma_{t_p} + \sigma_{t_T}\right\}\left\{\sigma_{\ell_L} + \sigma_{\ell_T}\right\}\right]_{r=r_2} + \left\{\sigma_{\ell_L} + \sigma_{\ell_T}\right\}^2_{r=r_2}\end{aligned} \qquad \text{Eq. (17)}$$

Expanding and observing that tangential and longitudinal thermal components are identical in magnitude at a given element and that the axial-load stress is the same for all elements within the wall:

$$\begin{aligned}\sigma_{\ell_L}\,{}_{r=r_1 \text{ and } r_2} = &\left\{\sigma_{t_p}^2 + \sigma_{t_T}^2 + \sigma_{t_p}\,\sigma_{t_T}\right\}_{r=r_2} - \left\{\sigma_{t_p}^2 + \sigma_{t_T}^2\right. \\ &\left. + \sigma_{t_p}\,\sigma_{t_T}\right\}_{r=r_1} \div \left\{\sigma_{t_T} - \sigma_{t_p}\right\}_{r=r_1} - \left\{\sigma_{t_T} - \sigma_{t_p}\right\}_{r=r_2}\end{aligned} \qquad \text{Eq. (18)}$$

Thus, when the resultant axial load from air and water pressure produces the above value of longitudinal stress, the working stress, σ_w[†], becomes a minimum. For given geometry and tunnel stagnation pressure, this may be accomplished by the judicious choice of cooling water pressure and liner flange diameter.

[†]Working stress, as used throughout this report, is taken to be the combination, by means of the shear-distortion theory of failure, of all resultant tangential and longitudinal stresses.

Now consider the portion of the thermal balance identity:

$$h_a \pi d_1 \Delta \ell (T_r - T_a) = \frac{\bar{K}}{t} \pi d_m \Delta \ell (T_a - T_w) \qquad \text{Eq. (19)}$$

Or, rewriting:

$$\frac{T_a - T_w}{t} = \frac{h_a d_1 (T_r - T_a)}{\bar{K} d_m} \qquad \text{Eq. (20)}$$

When evaluated, this equation yields the approximate temperature drop per inch of wall thickness which will satisfy a thermal balance; for any particular thickness, then, a discrete temperature drop is implicit and subsequently enters into the calculation of thermal stress. Upon substitution of various values of wall thickness and performing the calculations in Eqs. (1) - (16), a plot of working stress versus thickness may be constructed. As demonstrated later, an optimum is evident.

4.3 SELECTION OF THE COOLING REQUIREMENTS

General considerations governing the selection of the cooling requirements are:

(a) The selected water flow rate and cooling passage geometry must be such that the air-side-wall temperature and the thermal gradient through the wall remain compatible with the elevated temperature properties of the liner material.

(b) The water supply system must be capable of supplying the required flow rate through the selected cooling passage.

Now, if it is assumed that the conditions of (a) have been fulfilled, the required wall-to-water heat transfer coefficient may be determined. Many combinations of cooling passage geometry and flow rate will result in the required coefficient, and, although no optimum combination is apparent, analysis indicates that the smaller the water annulus, the lesser total head to produce the required flow. However, it must be realized that a lower limit exists because of the practical aspects of fabrication and assembly; also, a smaller passage implies a system more sensitive to the minor variations encountered in machining or in the flow control devices. Therefore, an area that is commensurate with the capabilities of the pumping system on hand (or under consideration) should be used.

4.4 HEAT TRANSFER

Calculation of the air-to-cooled-wall heat transfer coefficient at the throat and downstream may be accomplished by means of the relationship (Ref. 3):

$$h_a = St\ g\ c_p\ V_s \qquad \text{Eq. (21)}$$

Where

$$St = (C_f/2)\ Pr^{-2/3} \qquad \text{Eq. (22)}$$

The skin friction coefficient, C_f, may be derived by the method of Sivells and Payne (Ref. 4), viz.:

$$C_f = \frac{T_s\,(0.088)\,(\log_{10} Re - 2.3686)}{T'\,(\log_{10} Re - 1.5000)^3} \qquad \text{Eq. (23)}$$

In the above, primed values refer to conditions at Eckert's reference temperature, $T' = \frac{T_a + T_s}{2} + 0.03941\ M^2\ T_s$; this means, then, that the heat transfer coefficient and air-side-wall temperatures are interdependent, and the correct value for either results only from an iterative process. Though this may appear laborious, in actuality it is not because a specific air-side-wall temperature, selected as the maximum desirable for the material under consideration, must be met at the throat. The computation is therefore direct at this point, and if a thermal balance is calculated at two or three downstream locations, intermediate points may be readily estimated and checked. Upstream of the throat, the coefficient is taken to be simply:

$$h_a = \left\{\frac{A^*}{A}\right\}^{0.875} \times h^* \qquad \text{Eq. (24)}$$

5.0 SPECIFIC ANALYSIS

From the general method of analysis outlined in sections 4.2, 4.3, and 4.4, it is apparent that a near-infinite number of solutions is possible, each dependent on particular assumptions which must be made at various stages of the calculation. These assumptions, however, are arbitrary, provided they fulfill the requirements of practicability in manufacturing and assembly and are shown to be realized at a subsequent stop in the analysis. In the following subsections, assumptions are therefore made without argument as to any fictitious standard of "best possible" value.

Listed below are the salient design parameters for the Mach 10 and 12 throat sections:

	M = 10	M = 12
p_O, psia	2000	2400
T_O, °F	1450	1940
Throat radius, r*, in.	0.873	0.560
Throat station, downstream of the pressure housing entrance, in.	20.571	18.704
Radius ratio, R*/r*	30.000	38.179

Mechanical and physical properties of the liner material, as given in Ref. 5, are plotted in Fig. 3. It should be noted that all values result from short-time tests at elevated temperature, and a liberal margin of safety must therefore be provided. Consequently, a maximum working stress of 60,000 psi (i. e., a 33-1/3 percent margin on the proportional limit) and a limit of 650°F on the air-side wall will be used. As the material may be age-hardened after machining, and this process will continue during operation, the mean conductivity may be chosen as conservative; versus temperature, the relationship (see Fig. 3) is:

$$\bar{K} = \left\{\frac{(484)(T_a + T_w)}{(1000)(2)} + 134\right\}, \frac{\text{Btu ft}}{\text{hr ft}^2\ {}^{\circ}\text{F}} \times \frac{\text{ft}}{12\text{ in.}} \times \frac{\text{hr}}{3600\text{ sec}}$$

$$= \frac{0.0242\,(T_a + T_w) + 134}{43{,}200}, \frac{\text{Btu in.}}{\text{sec in.}^2\ {}^{\circ}\text{F}}$$

$$= \frac{T_a + T_w + 5535}{1{,}784{,}000}, \frac{\text{Btu in.}}{\text{sec in.}^2\ {}^{\circ}\text{F}} \qquad \text{Eq. (25)}$$

Other material properties, again from Ref. 5, are:

$$E = 17.5 \times 10^6 \text{ psi}, \ \alpha = 9.8 \times 10^{-6} \frac{\text{in.}}{\text{in. }{}^{\circ}\text{F}}, \ \bar{\nu} = 0.3$$

Air and water properties, as given in Refs. 6 and 7, respectively, are plotted versus temperature in Figs. 4 and 5.

In the following subsections, only the downstream portion of the liner will be considered; analysis of the less critical upstream part may be accomplished by the same method but is not included herein.

5.1 MACH 10

A cursory analysis of Eq. 10 indicates that T_a at the throat may sonably be held to 600°F provided h_a does not exceed 1.5 Btu/sec ft^2 By the Sibulkin equation (Ref. 8):

$$h_a = \frac{0.0027\ p_o\ \nu^{*\,0.2}}{T_o^{\,0.6}\ (r^*\ R^*)^{0.1}} \left\{\frac{T_1}{T_{\frac{1+w}{2}}}\right\}^{*\dagger}$$

Substituting pertinent values and dimensional constants:

$$h_a = \frac{(0.0027)(2000\ \times\ 144)(0.811\ \times\ 10^{-6}\ \times\ 18.18)^{0.2}}{(1935)^{0.6}\ (\overline{.07275^2}\ \times\ 30)^{0.1}} \frac{(1612)}{(1336)} = 1.30\ \frac{\text{Btu}}{\text{sec ft}^2\ {}^\circ\text{F}}$$

Now consider Eq. (20) which is written:

$$\frac{T_a - T_w}{t} = h_a\ \frac{d_1\,(T_r - T_a)}{\overline{K}\ d_m}$$

Assume $d_1 = d_m$ and $T_r = T_o$; $\overline{K}$ at 600°F is 163 Btu ft/hr ft^2 °F (see Fig. 3). Using, for conservatism, the larger value of h_a and substituting:

$$\frac{T_a - T_w}{t} = \frac{(1.5)(1475 - 600)(3600)}{(163)(12)} = 2420\ \frac{{}^\circ\text{F}}{\text{inch of wall}}$$

Consider, next, the axial load at the throat produced by the air. I a telescoping joint is to be provided where the Mach number is 0.1, th local ordinate becomes:

$$\left(\frac{r}{r^*}\right)^2 = 5.8218,\quad \therefore\ r = 0.873\ \sqrt{5.8218} = 2.013\ \text{in.}$$

The maximum axial load is:

$$L_p = \pi\,(r^2 - r^{*2})\,p$$

$$= \pi\,(3.68)(2000) = 23{,}300\ \text{lb}$$

It is expedient stresswise to counterbalance a large portion of the abov and this may be accomplished by cooling water pressure acting over th face of a flange made integral with the liner. A rational combination is an 8-inch flange and 400-psig water pressure; this produces about 19, 000 lb of opposing load.

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† At the time the ensuing calculations were performed, T_o was considered to be 1475°F. This was later reduced to the previously men tioned value of 1450°F, thereby reflecting differences between a perfect and a real gas. A small element of conservatism is thus inherent.

The working stress may now be calculated for various wall thicknesses by substituting previously mentioned values of E, α, $\bar{\nu}$, r, p_o, p_w, and $\Delta T/t$ into the applicable portion of the Eqs. (1) - (16); see Table 1 and Fig. 6. It should be noted that an optimum exists at a thickness of 0.057 inch; this, however, is less than the practical machining range, and, since an increase to 0.125 inch produces a stress still well below the selected allowable, the latter value will be used as the nominal throat wall.

Now, consider the required wall-to-water heat transfer coefficient. Assume T_a and h_a as before and let the bulk temperature of the water be 60°. T_r, from Eq. (11), = 1612 + 0.89 (1935 - 1612) = 1900°R. Then, for a 0.125-inch wall, $T_w = T_a - \Delta T/t \times t = 600 - (2420)(0.125) \cong 300°F$. From the portion of the thermal balance identity written:

$$h_a\, r_1\, (T_r - T_a) = h_w\, r_2\, (T_w - T_b)$$

$$h_w = h_a \frac{r_1 (T_r - T_a)}{r_2 (T_w - T_b)}$$

Substituting:

$$h_w = \frac{(1.5)(0.873)(1440 - 600)(3600)}{(0.998)(300 - 60)} \cong 16{,}510 \ \frac{\text{Btu}}{\text{hr ft}^2\ {}^\circ\text{F}}$$

From Eq. (13):

$$h_w = \frac{(0.023)\, c_p^{1/3}}{\nu_w^{0.14}} \left\{ \frac{\bar{K}^{2/3}}{\nu^{1/3}} \right\} \frac{G^{0.8}}{D^{0.2}} = 16{,}510$$

Solving:

$$\frac{G^{0.8}}{D^{0.2}} = \frac{(16{,}510)(0.886)}{(0.023)(0.3500)} \cong 1.817 \times 10^6$$

Reducing the above by means of the proper conversion units and the relationship $D = 2(r_3 - r_2)$:

$$X^{0.8} = 390\,(r_3 - r_2)(r_3 + r_2)^{0.8}, \quad \text{gpm}^{0.8}$$

Solution for any particular annulus height yields a discrete flow which will limit the air-side temperature to 600°. Calculations have been made for several passages, and the results, including total head requirements†, are given in Table 2 and plotted in Fig. 7. The latter also shows the operating characteristics of a high pressure pump on hand in the von Kármán Facility and indicates that, as stated previously, no optimum combination exists. Flow of 275 gpm through a 0.125-inch annulus, however, will meet the general requirements outlined in

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†Pressure drop for the entire section is estimated at 4 times that per inch of passage on the throat.

section 4. 3 and will have the advantage that, if during operation, temperatures rise above those calculated, flow may be considerably increased without exceeding the pump system capabilities. This, of course, increases the thermal stress which, for radial heat flow in a cylindrical section, is, roughly:

$$\sigma_T = \frac{1}{2}\,\frac{E\alpha\Delta T}{(1-\bar{\nu})}, \quad \therefore \quad \frac{\sigma_T}{\Delta T} \cong \frac{1}{2}\,\frac{(17.5)(10)}{(0.7)} \cong 125 \ \frac{\text{psi}}{{}^{\circ}\text{F}}$$

Substituting:

$$\sigma_T = \frac{(17.5)(9.8)(300)}{(1.4)} \cong 37{,}500 \ \text{psi}$$

The above will account for approximately two-thirds of the working stress. Thus for an allowable 60, 000 psi:

$$\Delta T = \frac{60{,}000 - (1.33)(37{,}500)}{(125)} \cong 80^{\circ}$$

= the approximate allowable increase in thermal gradient.

Before the heat transfer coefficient and thermal balance can be calculated along the length of the liner, the variation in wall thickness should be stated. This is quite arbitrary provided temperatures may subsequently be shown to decrease smoothly and continuously each way from the throat. Assuming, therefore, a constant wall ratio and arranging the equations of sections 4. 1, 4.3, and 4. 4 in computational form, Tables 3 and 4 may be readily prepared. Constants shown in the column headings are explained at the start of the appendix. Assumed parameters, in summary, are:

a) $T_a = 600\ {}^{\circ}F$

b) $X = 275$ gpm

c) $(r_2 - r_1)^* = 0.125$ in.

d) $(r_3 - r_2) = 0.125$ in., constant

e) $K = (r_2/r_1)^* = 1.1455$, constant

f) M and r_1 = values given by the aerodynamic contour † and listed in Column 1 of Tables 3 and 4, respectively.

g) The design parameters as listed in section 5.0.

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† For the method used in determining the aerodynamic contour, see Ref. 9.

The results are plotted in Fig. 8. Then, following the form of Table 1 but omitting the calculations:

$$\sigma_w = +33{,}600 \text{ psi at } r = r_2, \quad \sigma_w = -37{,}300 \text{ psi at } r = r_1$$

Because the above stresses do not exceed the selected allowable value, and the value of T_a calculated in the thermal balance does not differ by more than 3 deg from that used in determining h_a, a reasonable and workable solution has been found.

5.2 MACH 12

Because the method of analysis for Mach 12 is identical to that for Mach 10, only the final results and significant differences resulting from the higher stagnation conditions are discussed herein.

The minimum practical value of T_a, from the standpoint of cooling requirements and resulting thermal gradient, is 650°F. This requires a flow of 700 gpm and reduction of the throat wall to 0.100 inch, about the minimum for practical machining. The annular passage at that location is chosen as 0.130 inch and, to reduce the overall frictional pressure drop, is rapidly increased either way from the throat. The wall ratio is not kept constant but varies in a manner which will facilitate interchangeability with the Mach 10 liner in a common pressure housing. Because of the required high water velocity at the throat (see Col. 55, Table 8), the cooling water pressure is determined as follows:

From Eq. (14)

$$p = \frac{V^2\rho}{2g} + \Delta p_f$$

Substituting

$$p = \frac{(379.2)^2(62.4)}{2(32.17)(144)} + 113 = 1081 \text{ psig}$$

To prevent cavitation, it is necessary to add to the above the vapor pressure of water at the temperature $T_w{}^* = 235°F$, which is about 8 psig. Also, some factor, say five, must be introduced on this value as well as an allowance for inaccuracy of the friction coefficient used in the pressure drop calculation. The summation is therefore:

Velocity head	969 psig
Pressure drop, entrance to throat	113 "
Prevention of cavitation X factor of 5 . .	40 "
25 percent error in pressure drop. . . .	28 "
	1150 psig

The above value of pressure should appear at the nozzle section flange, and the outlet pressure of the pump should be sufficient to cover line and entrance-to-nozzle section losses, say 1200 psig. The flange which is used to counterbalance the air pressure load is taken to be 4.75 inches in diameter. Substituting pertinent values into the applicable stress equations:

$$\sigma_W = -45{,}800 \text{ psi at } r = r_1^*, \quad \sigma_W = 56{,}000 \text{ psi at } r = r_2^*$$

The liner must also operate for the tunnel-air-off, cooling-water-pressure-on case; in this event, the stress is simply tension in amount:

$$\sigma_W = \frac{1150\,(2.375^2 - 0.66^2)}{(1.22)(0.1)} = 49{,}100 \text{ psi}$$

6.0 CONCLUDING REMARKS

The preceding pages adequately demonstrated the technical feasibility of the throat section designs shown in Figs. 1 and 2, and it remains, therefore, only to investigate the effect of deviation from the selected geometric and operational quantities. This will be of greatest consequence in the Mach 12 configuration to which the following remarks, except where noted, are accordingly confined.

A decrease in p_O or T_O poses no problem, and by inspection, a reasonable, say ±0.002-inch, machining tolerance on the liner ordinates will be of little structural consequence. The results of variations in the cooling flow rate or the annular passage size, however, are somewhat obscure. Consider, therefore, a ±10% change in the fundamental variable, h_W, which, if substituted into the heat transfer and thermal balance tabulations, yields the following values.

	No change (Ref.)	10% increase in h_W	10% decrease in h_W
h_a, $\frac{\text{Btu}}{\text{sec ft}^2\ ^\circ\text{F}}$	1.901	1.912	1.895
T_a, °F	654	644	664
T_w, °F	235	222	252
$T_a - T_w$, °F	419	422	412

Now, $h_w \approx G^{0.8}/D^{0.2}$, $G \approx 700$ gpm and $D_{nominal} = \frac{2\,(0.13)}{12}$, or 0.02167 feet. If h_W is to increase 10 percent:

a) $G \approx (700)(1.1)^{1.25}$ 788 gpm = 88 gpm increase, or,

b) $D = \left(\frac{1}{1.1}\right)^5 (0.02167) = 0.0128 \text{ ft} = 0.1545 \text{ in.}$

Therefore, $r_3 - r_2 = 0.077$ inch, a decrease of 0.067 inch. Similarly, if h_w is to decrease 10 percent:

a) $G = (700)(0.9)^{1.25}$ 614 gpm = 86 gpm decrease, or,

b) $D = \left(\frac{1}{0.9}\right)^5 (0.02167) = 0.0367 \text{ ft} = 0.4406 \text{ in.}$

Therefore, $r_3 - r_2 = 0.2203$ inch, an increase of 0.090 inch. Considerably less variation in annulus height than the above may be expected from careful machining and assembly techniques. It is, in fact, quite reasonable to stipulate a passage of 0.130 inch minimum, plus a tolerance, say 10 percent, which is 0.013 inch. Also, because pressure requirements go up as the square of an increase in flow, it is more probable that such deviation will tend toward a drop from the nominal. Then, if h_w is to decrease 10 percent:

$$0.9 = \frac{G^{0.8}}{1.1^{0.2}}, \quad \therefore G = (700)\left\{(0.9)(1.1)^{0.2}\right\}^{1.25} = 628 \text{ gpm}$$

It is felt that the above is within practical flow control limits, and the accompanying rise to 664°F of the air-side wall is not serious, primarily because of the conservatism exercised in choosing the metal conductivity. One final variable, the bulk temperature of the cooling water, remains to be considered. Briefly, for an increase from 60 to 80°F, recomputation of the throat conditions indicates no significant change in h_a or T_a and only a 3-degree rise in T_w.

Finally, consider the liner flange which, for the Mach 10 configuration in particular, encompasses a heated area somewhat removed from the cooling water. By means of a thermal balance, the temperature of a thermocouple located within the flange (Fig. 1) may be conservatively approximated as 800°F. This, of course, exceeds the selected allowable value, but it should be noted that pressure stresses are essentially absent, and no account has been taken of the low temperature air which spills past the heated surface. Accordingly, the calculated temperature is acceptable.

Two comments may now be offered in partial support of the adequacy of the design analysis as discussed herein. First, as was mentioned previously, the Mach 10 version is currently in operation and has successfully accumulated a total air-on time in excess of 1050 hours. Secondly, the thermocouple discussed above indicated an initial temperature of 625°F during operation at near-maximum stagnation conditions; subsequently, upon build-up of a so-far-harmless scale deposit, it decreased, stabilizing at about 525°F.

In conclusion, the Mach 12 liner design is specifically proportioned to effect a lowering of the above-mentioned temperature to a calculated 500°F; in addition, a test program is currently underway to select a serviceable plating material which will inhibit the formation of scale. The test method and results to date are given in Ref. 1.

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APPENDIX I

DERIVATION OF CONSTANTS FOR THE HEAT TRANSFER COEFFICIENT AND THERMAL BALANCE CALCULATIONS

a) 0.03491 in Col. 8, Tables 3 and 7 - constant in Eckert's reference temperature, see Eq. (31) Ref. 4.

b) 53.3 in Col. 10, Tables 3 and 7 - the gas constant for air in ft/°R.

c) 4.476 in Col. 11, Tables 3 and 7 - conversion from lb/in.^{2} ft to slugs/ft^3.

d) 49.1 in Col. 13, Tables 3 and 7 - from the basic aerodynamic relationship, $v = 49.1\ M\sqrt{T}$.

e) 15.3627 in Col. 15, Table 3 - the throat abscissa minus a reference distance, $x^* = \sqrt{\left[(\gamma+1)/2\right] r^*R^*}$, (Ref. 4) in which $\gamma = 1.373$ (Ref. 5), and $R^*/r^* = 30.000$.

f) 14.9522 in Col. 15, Table 7 - the throat abscissa minus a reference distance, $x^* = \sqrt{\left[(\gamma+1)/2\right] r^*R^*}$, (Ref. 4) in which $\gamma = 1.352$ (Ref. 5), and $R^*/r^* = 38.1786$.

g) 144 in Col. 19, Tables 4 and 8 - conversion from ft^2 to in.^{2}

h) 1,784,000 in Cols. 22 and 23 and 5535 in Cols. 24 and 40, Tables 4 and 8 - from Eq. (25) for average conductivity at the average of the air-and-water-side-wall temperatures.

i) 393,525 in Col. 25, Table 4 - from the portion of the Colburn relation which is written $G^{0.8}/D^{0.2}$. If the flow is 275 gpm:

$$G = 275\ \frac{\text{gal}}{\text{min}} \times 60\ \frac{\text{min}}{\text{hr}} \times \frac{\text{ft}^2}{7.48\ \text{gal}} \times 62.4\ \frac{\text{lb}}{\text{ft}^3} \times \frac{144\ \text{in.}^2}{\pi(r_3^2 - r_2^2)\,\text{in.}^2\,\text{ft}^2}$$

$$= \frac{6{,}310{,}000}{(r_3 + r_2)(r_3 - r_2)},\ \frac{\text{lb}}{\text{hr ft}}$$

$$D = 2(r_3 - r_2)\ \text{in.} \times \frac{\text{ft}}{12\ \text{in.}}$$

$$\frac{G^{0.8}}{D^{0.2}} = \frac{6{,}310{,}000^{0.8} \times 6^{0.2}}{(r_3 + r_2)^{0.8}(r_3 - r_2)} = \frac{393{,}525}{(r_3 + r_2)^{0.8}(r_3 - r_2)}$$

j) 833,000 in Col. 25, Table 8 - from the portion of the Colburn relation which is written $G^{0.8}/D^{0.2}$. If the flow is 700 gpm:

$$G = 700 \frac{gal}{min} \times 60 \frac{min}{hr} \times \frac{ft^3}{7.48\,gal} \times 62.4 \frac{lb}{ft^3} \times \frac{144\ in.^2}{\pi\,(r_3^2 - r_2^2)\ in.^2\ ft^2}$$

$$= \frac{16{,}060{,}000}{(r_3 + r_2)(r_3 - r_2)},\ \frac{lb}{hr\ ft}$$

$$D = 2\,(r_3 - r_2)\ in. \times \frac{ft}{12\ in.}$$

$$\frac{G^{0.8}}{D^{0.2}} = \frac{16{,}060{,}000^{0.8} \times 6^{0.2}}{(r_3 + r_2)^{0.8}\,(r_3 - r_2)} = \frac{833{,}000}{(r_3 + r_2)^{0.8}\,(r_3 - r_2)}$$

k) 0.023 in Col. 23, Tables 4 and 8 - a constant in the Colburn equation.

l) 518,400 in Col. 30, Tables 4 and 8 - conversion from ft^2 hr to $in.^2$ sec.

m) 38.23 in Col. 53, Table 4 - from the relationship $q = Wcp\Delta T$ if the flow rate is 275 gpm.

$$\Delta T = q \frac{Btu}{sec} \times \frac{min}{275\ gal} \times 60 \frac{sec}{min} \times 7.48 \frac{gal}{ft^3} \times \frac{ft^3}{62.4\ lb} \times \frac{lb\ ^\circ F}{1\ Btu}$$

$$= \frac{q,\ ^\circ F}{38.23}$$

n) 97.35 in Col. 53, Table 8 - from the relationship $q = Wcp\Delta T$. If the flow rate is 700 gpm:

$$\Delta T = q \frac{Btu}{sec} \times \frac{min}{700\ gal} \times 60 \frac{sec}{min} \times \frac{7.48\,gal}{ft^3} \times \frac{ft^3}{62.4\ lb} \times \frac{lb\ ^\circ F}{1\ Btu} = \frac{q,\ ^\circ F}{97.35}$$

o) 88.22 in Col. 55, Table 4 - from the relationship, Q = AV. For 275 gpm flow:

$$V = 275 \frac{gal}{min} \times \frac{min}{60\ sec} \times \frac{ft^3}{7.48\ gal} \times \frac{1}{Area,\ in.^2} \times \frac{144\ in.^2}{ft^2}$$

$$= \frac{88.22}{Area,\ in.^2},\ \frac{ft}{sec}$$

p) 224.56 in Col. 55, Table 8 - from the relationship, Q = AV. For 700 gpm flow:

$$V = 700 \frac{gal}{min} \times \frac{min}{60\ sec} \times \frac{ft^3}{7.48\ gal} \times \frac{1}{Area,\ in.^2} \times \frac{144\ in^2}{ft^2}$$

$$= \frac{224.56}{Area,\ in.^2},\ \frac{ft}{sec}$$

q) 15,000 in Col. 57, Tables 4 and 8 - portion of Re for water at an average temperature of, say 65°F:

$$\frac{\rho D}{\nu} = 62.4\ \frac{\text{lb}}{\text{ft}^3} \times 2\,(r_3 - r_2)\ \text{in.} \times \frac{\text{ft}}{12\ \text{in.}} \times \frac{\text{hr ft}}{2.50\ \text{lb}} \times 3600\ \frac{\text{sec}}{\text{hr}}$$

$$= 15{,}000\,(r_3 - r_2),\ \frac{\text{sec}}{\text{ft}}$$

r) 0.01371 in Col. 60, Tables 4 and 8 - from the portion of the friction coefficient equation $0.125/\text{Re}^{0.32}$. This may be written as

$$\frac{0.125}{\left\{\frac{\text{Re}}{1000}\right\}^{0.32}} \times \frac{1}{1000^{0.32}} = \frac{0.125}{\left\{\frac{\text{Re}}{1000}\right\}^{0.32}} \times \frac{1}{9.1} = \frac{0.01371}{\left\{\frac{\text{Re}}{1000}\right\}^{0.32}}$$

s) 0.01346 in Col. 62, Tables 4 and 8 - portion of the frictional drop equation, ρ/g, viz.:

$$\frac{\rho}{g} = 62.4\ \frac{\text{lb}}{\text{ft}^3} \times \frac{\text{sec}^2}{32.17\ \text{ft}} \times \frac{\text{ft}^2}{144\ \text{in.}^2} = 0.01346,\ \frac{\text{lb sec}^2}{\text{ft}^2\ \text{in.}^2}$$

APPENDIX II

DERIVATION OF CONSTANTS FOR THE OPTIMUM THROAT-THICKNESS CALCULATION

a) 122.5 in Cols. 10 and 13, Tables 1 and 5 - from the portion of the thermal stress equation, $E\alpha/(1 - \overline{\nu})$, psi/deg.

b) 2115, 400, 1057, and 800, Cols. 15, 16, 18, and 19, respectively, Table 1, 2000 psi air pressure or 400 psi water pressure times a constant in the applicable stress equation.

c) 2537, 500, 1268, and 1000, Cols. 15, 16, 18, and 19, respectively, Table 5 - as above, but for 2400 psi air pressure, 500 psi water pressure.

APPENDIX III

DERIVATION OF CONSTANTS FOR FLOW AND PRESSURE REQUIREMENT CALCULATIONS

a) 0.1021 in Col. 9, Tables 2 and 6 -

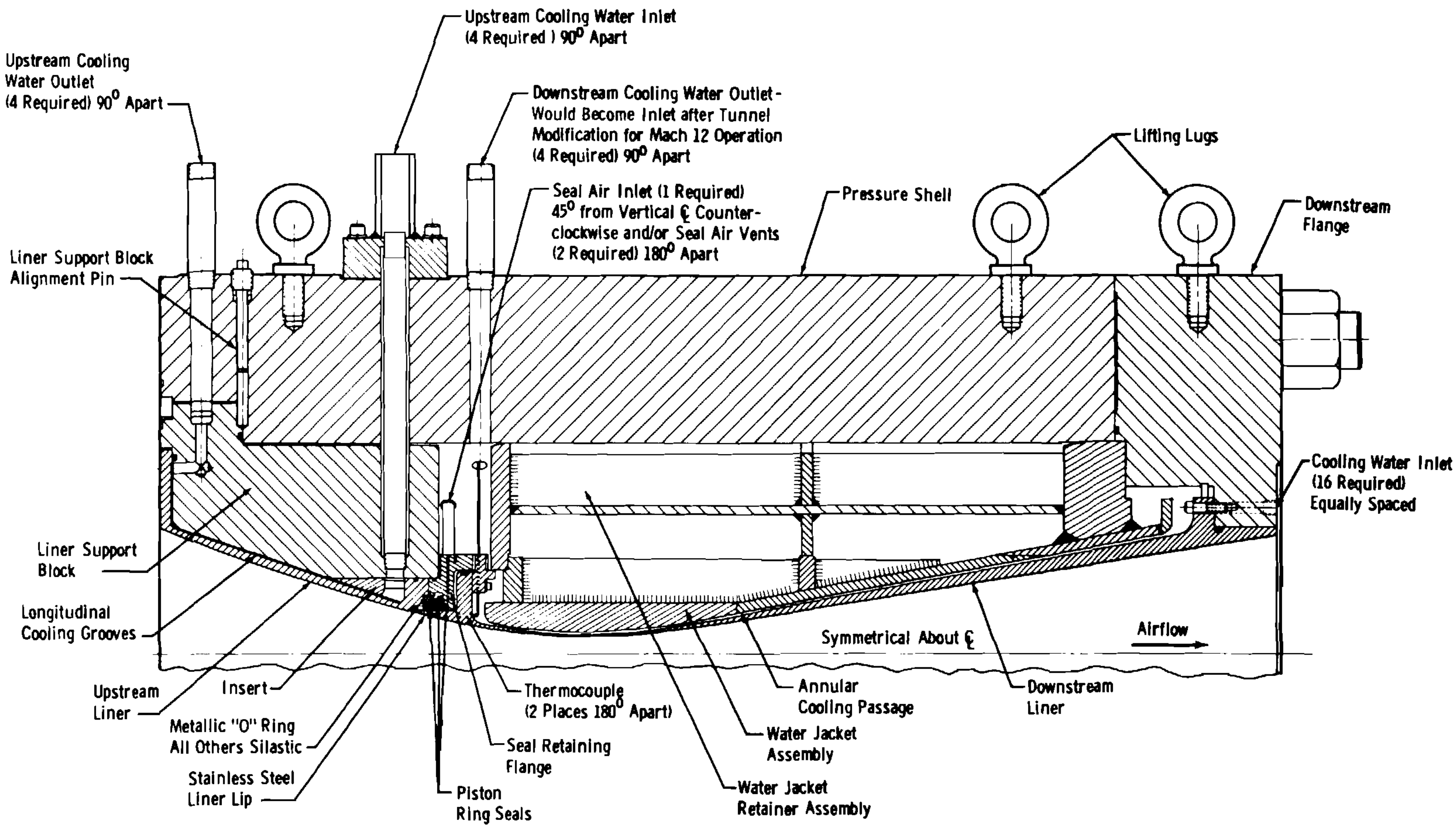
$$V = X \frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60 \text{ sec}} \times \frac{\text{ft}^3}{7.48 \text{ gal}} \times \frac{1}{\pi (r_3^2 - r_2^2) \text{ in.}^2} \times \frac{144 \text{ in.}^2}{\text{ft}^2}$$

$$= \frac{0.1021 X}{(r_3^2 - r_2^2)}, \frac{\text{ft}}{\text{sec}}$$

b) 0.433 in Col. 13, Tables 2 and 6 - Conversion of feet of water head to lb/in.^{2} pressure.

c) 4 in Col. 20, Table 2 - estimated total pressure drop per inch of throat drop.

d) 2.5 in Col. 20, Table 6 - Estimated total pressure drop per inch of throat drop.

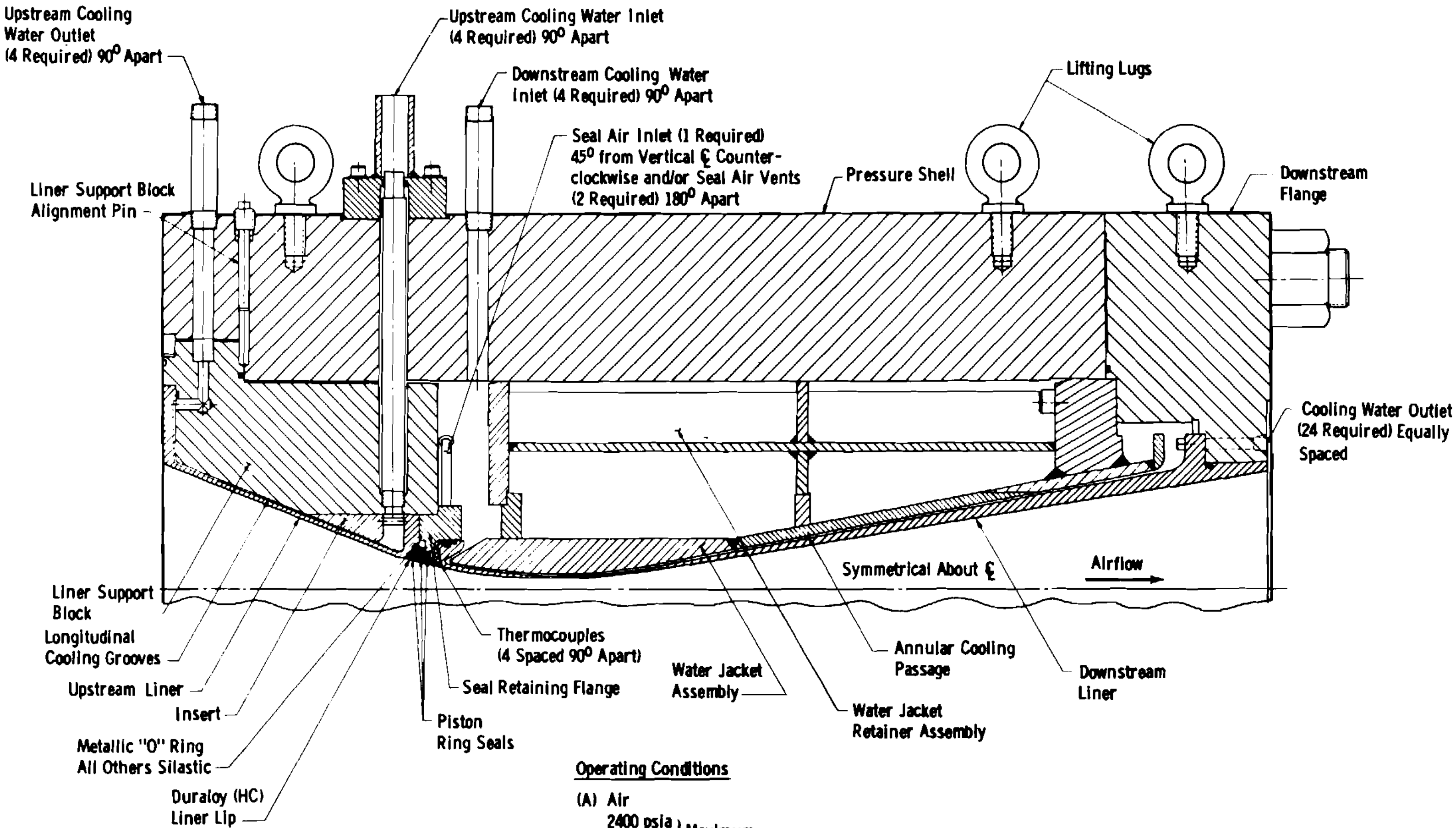


Operating Conditions

(A) Air
2000 psia } Maximum
1450 °F

(B) Cooling Water
50 gpm, 400 psig, Upstream
275 gpm, 400 psig, Downstream

Fig. 1 Mach 10 Throat Section



Operating Conditions

(A) Air
2400 psia } Maximum
1940 °F
(B) Cooling Water
50 gpm, 1150 psig, Upstream
700 gpm, 1150 psig, Downstream

Fig. 2 Mach 12 Throat Section

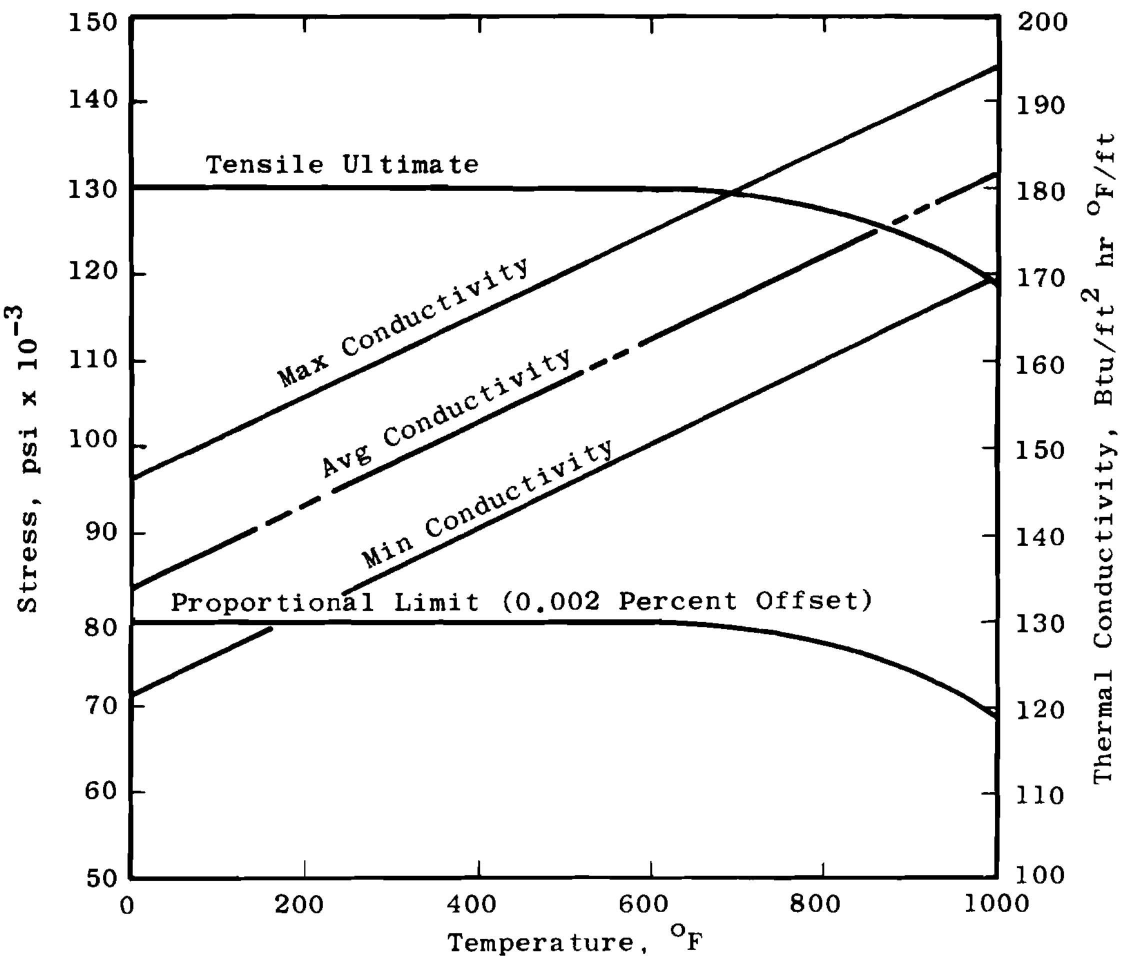


Fig. 3 Properties of Berylco-10 vs Temperature (Ref. 5)

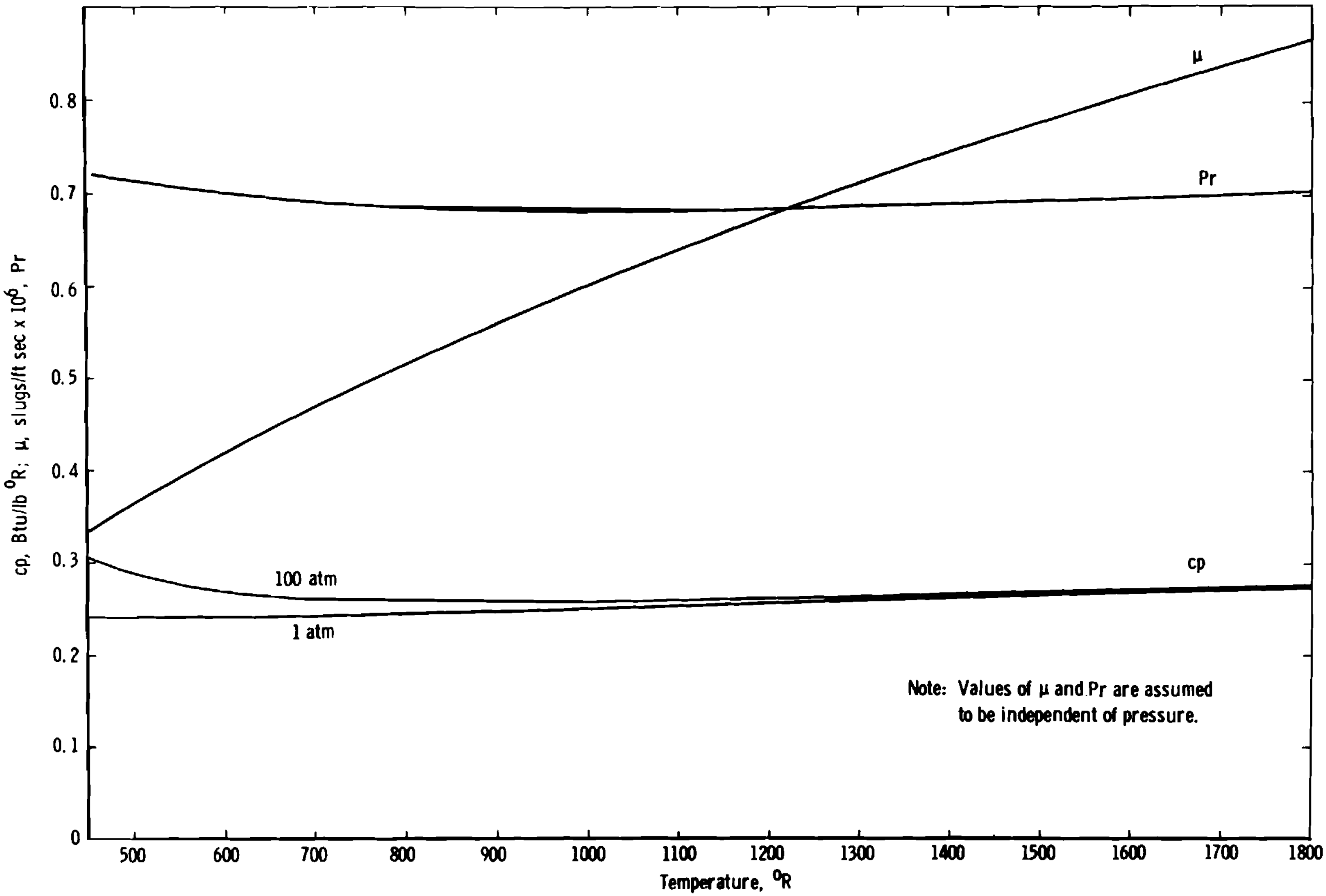


Fig. 4 Properties of Air vs Temperature (Ref. 6)

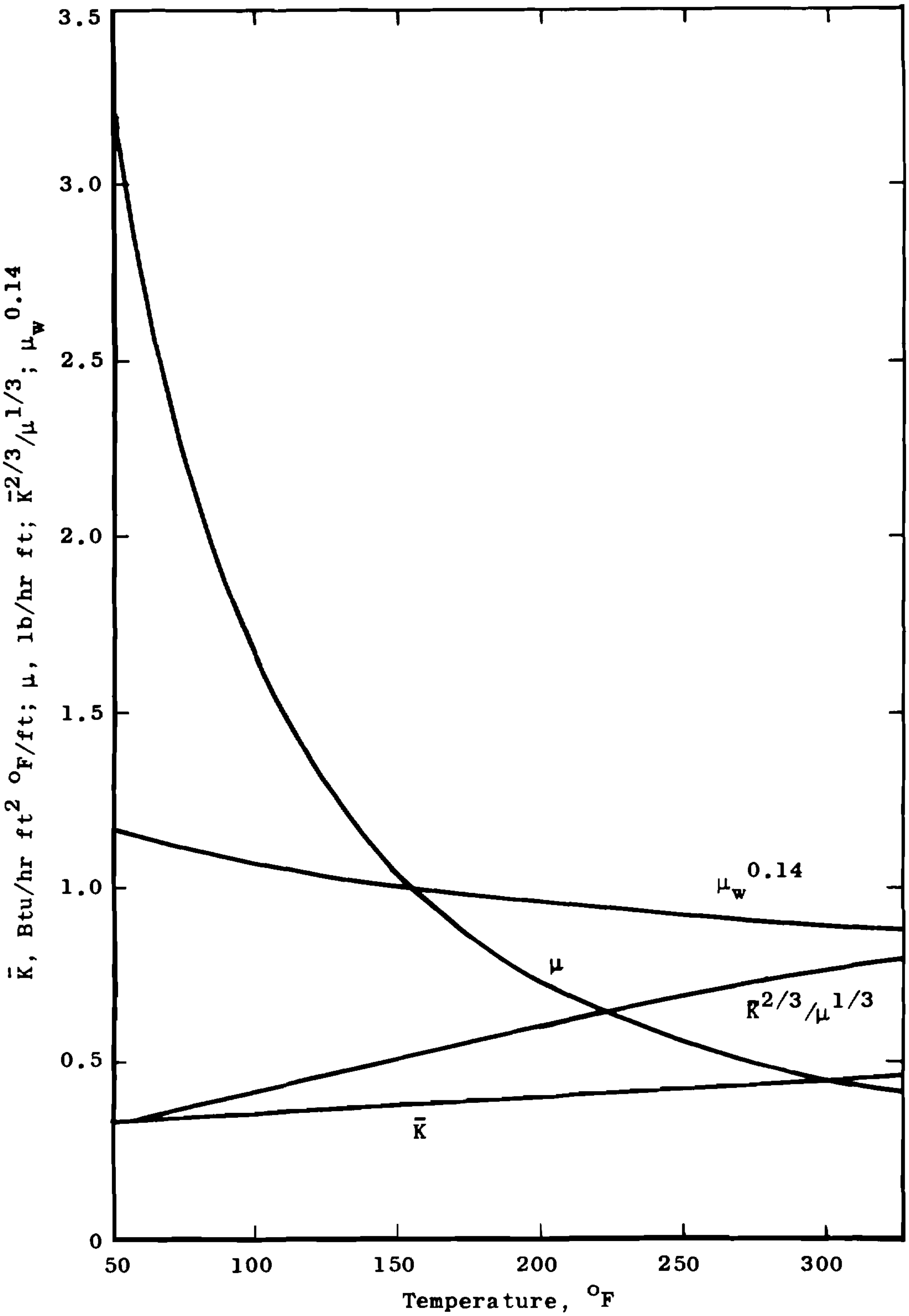


Fig. 5 Properties of Water vs Temperature (Ref. 7)

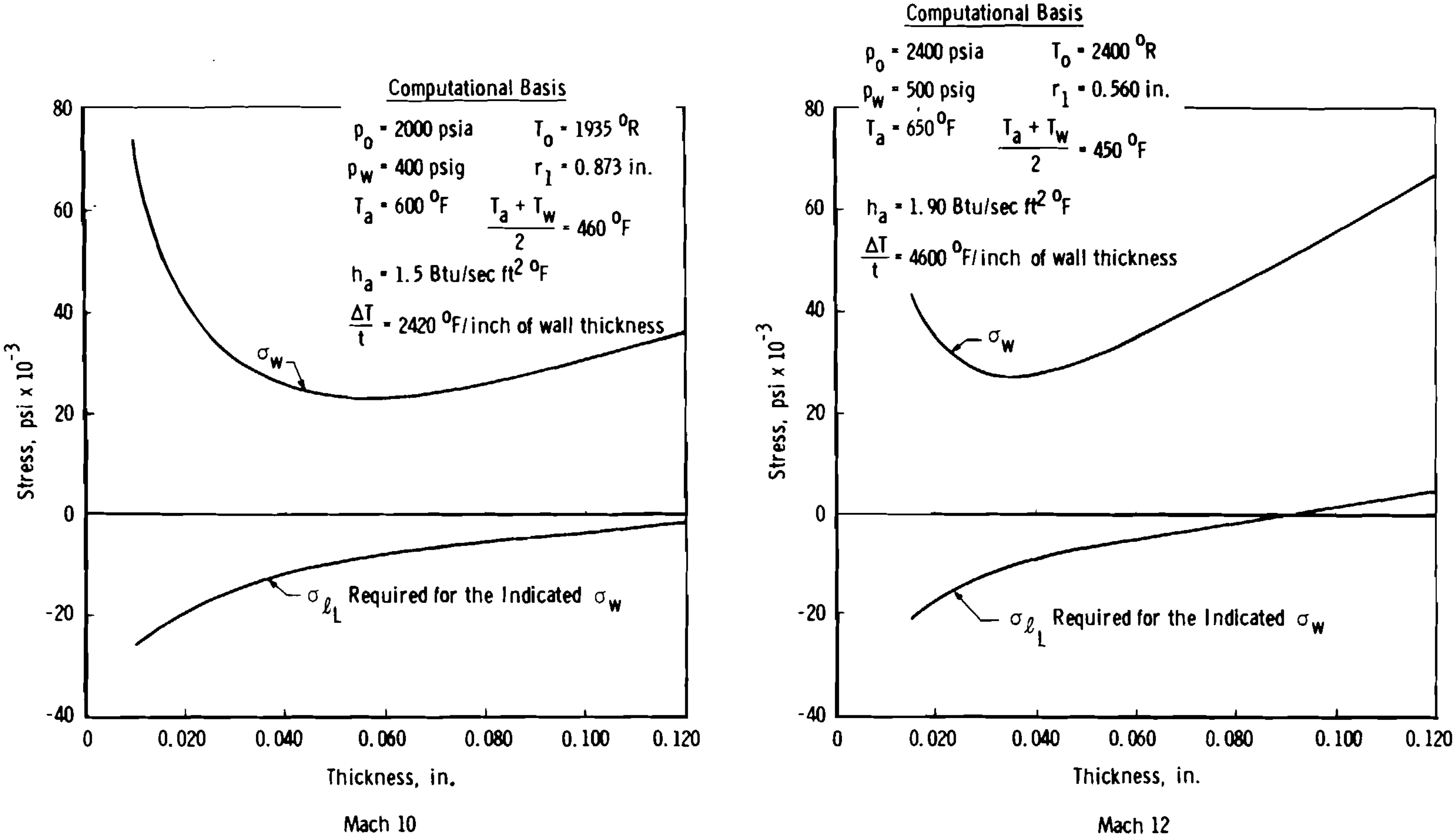


Fig. 6 Working Stress vs Wall Thickness of the Throat for Mach 10 and 12 Throat Sections

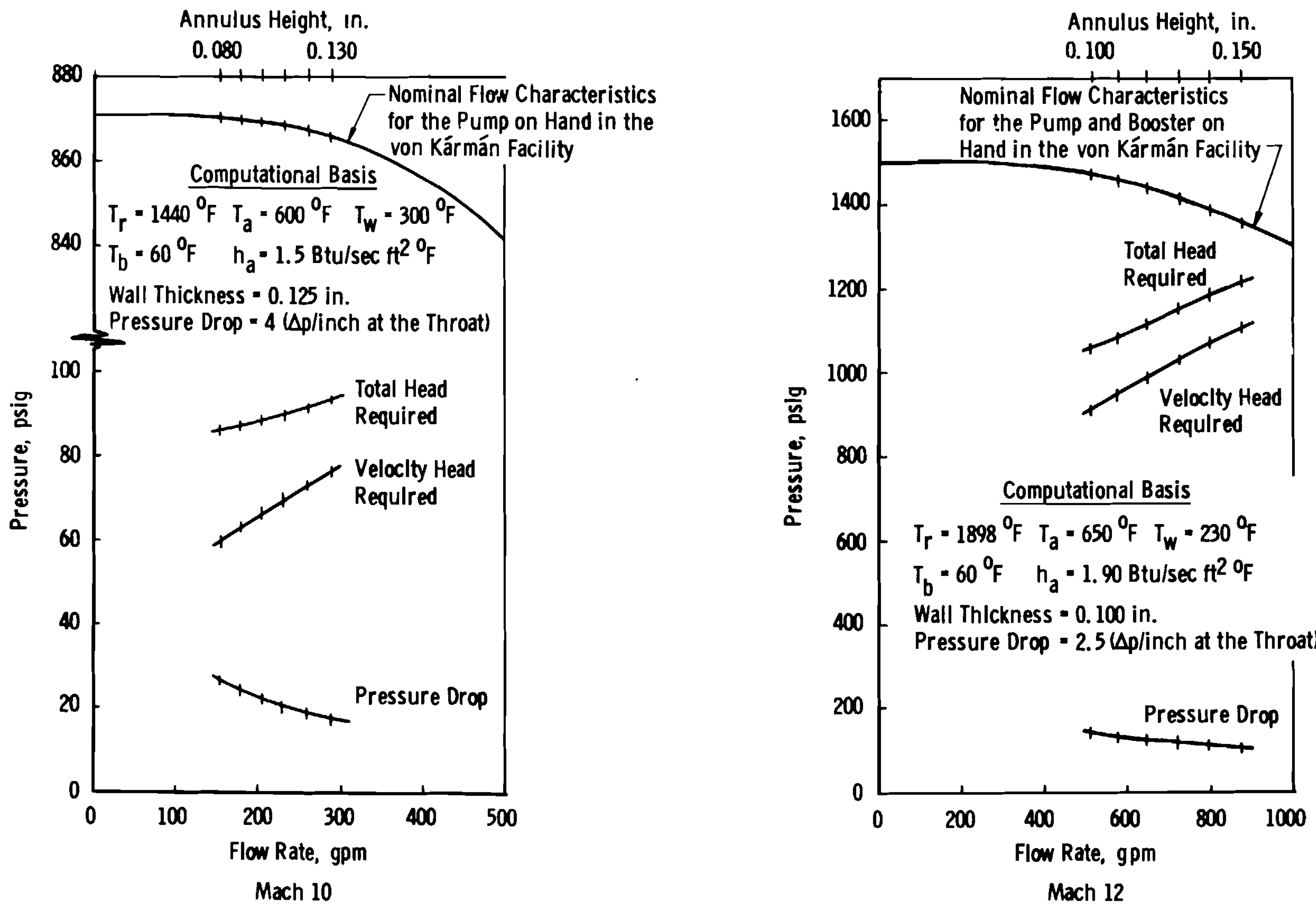


Fig. 7 Flow and Pressure Requirements for Various Annulus Heights, Wall Temperature Constant

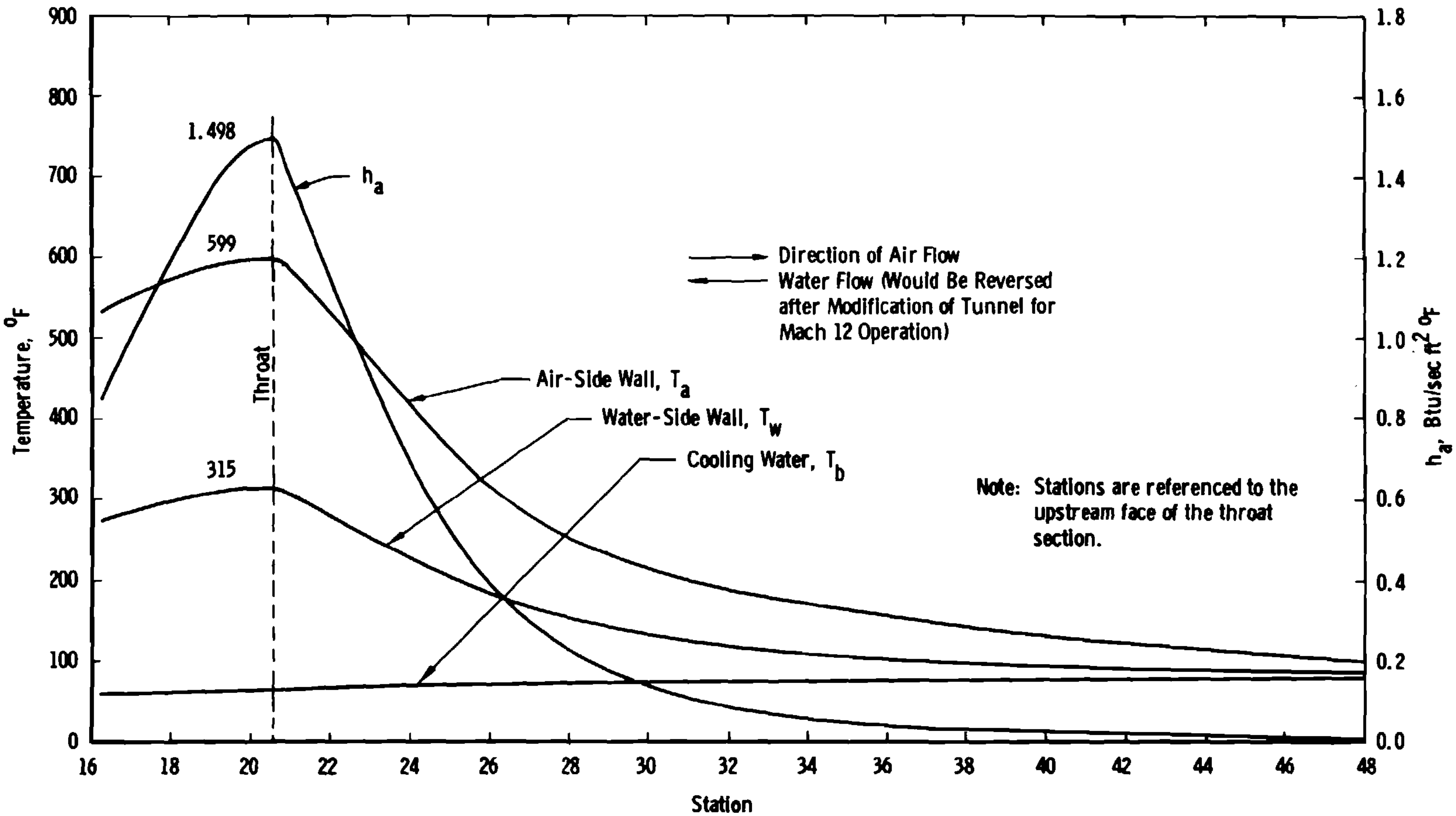


Fig. 8 Air-to-Wall Heat Transfer Coefficient and Temperature Distribution for Mach 10 Throat Section

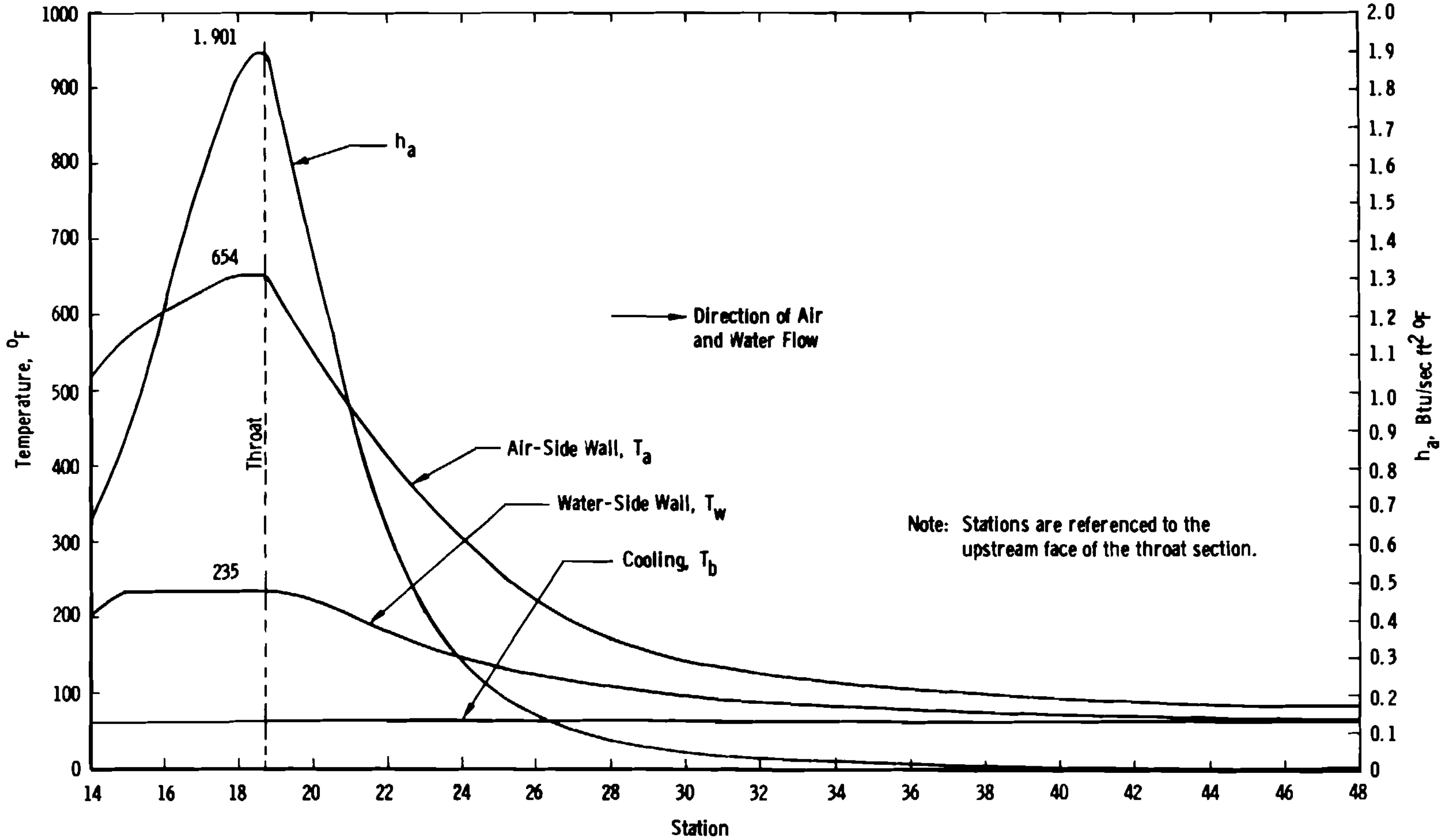


Fig. 9 Air-to-Wall Heat Transfer Coefficient and Temperature Distribution for Mach 12 Throat Section

TABLE 1

CALCULATION OF OPTIMUM THROAT THICKNESS OF THE MACH 10 THROAT SECTION

(Ref: Eqs. (1) – (9))

IN	1	2	3	4	5	6	7	8	9	10
	°F									psi/°F
t	ΔT	K	K^2	K^2+1	K^2-1	$\ln K$				$-\bar{\gamma}_a$
	$2420 \times t$	$\frac{r_1+t}{r_1}$	$(2)^2$	$(3)+1$	$(3)-1$		$\frac{2\times(6)}{(5)}$	$(3)\times(7)$	$-(1-(8))$	$\frac{122.5\cdot(9)}{(6)}$
0.02	48.4	1.0229	1.0463	2.0463	.0463	.022641	.9780	1.0233	.0233	126.1
0.03	72.6	1.0344	1.0700	2.0700	.0700	.033821	.9663	1.0339	.0339	122.8
0.04	96.8	1.0458	1.0937	2.0937	.0937	.044782	.9558	1.0454	.0454	124.2
0.05	121.0	1.0573	1.1179	2.1179	.1179	.055718	.9452	1.0566	.0566	124.4
0.06	145.2	1.0687	1.1421	2.1421	.1421	.066443	.9352	1.0681	.0681	125.6
0.08	193.6	1.0916	1.1916	2.1916	.1916	.087645	.9149	1.0902	.0902	126.1
0.10	242.0	1.1145	1.2421	2.2421	.2421	.108406	.8955	1.1123	.1123	126.9
0.12	290.4	1.1375	1.2939	2.2939	.2939	.128832	.8767	1.1344	.1344	127.8
0.14	338.8	1.1604	1.3465	2.3465	.3465	.148765	.8587	1.1562	.1562	128.6

IN	11	12	13	14	15	16	17	18	19	20
	psi		psi/°F		OUTSIDE (AIR)	PRES. (WATER)	STRESS (COMB.)	INSIDE (AIR)	PRES. (WATER)	STRESS (COMB.)
t	$-\sigma_{T_{r=r_1}}$		$\bar{\gamma}_b$	$\sigma_{T_{r=r_2}}$	psi	psi	psi	psi	psi	psi
	$(1)\times(10)$	$1-(7)$	$\frac{122.5\times(12)}{(6)}$	$(1)\times(13)$	$\frac{2115}{(5)}$	$\frac{400\times(4)}{(5)}$	$(15)-(16)$	$\frac{1057\times(4)}{(5)}$	$\frac{800}{(5)}$	$(18)-(19)$
0.02	6103	.0220	119.0	5760	45680	17680	28000	46720	17280	29440
0.03	8915	.0337	122.1	8860	30210	11830	18380	31260	11430	19830
0.04	12020	.0442	120.9	11700	22570	8940	13630	23620	8540	15080
0.05	15050	.0548	120.5	14580	17940	7190	10750	18990	6790	12200
0.06	18240	.0648	119.5	17350	14880	6030	8850	15930	5630	10300
0.08	24410	.0851	118.9	23020	11040	4580	6460	12090	4180	7910
0.10	30710	.1045	118.1	28580	8740	3700	5040	9790	3300	6490
0.12	37110	.1233	117.2	34030	7200	3120	4080	8250	2720	5530
0.14	43570	.1413	116.4	39440	6100	2710	3390	7160	2310	4850

IN	21	22	23	24	25	26	27	28	29
	$\sigma^2_{T_{r=r_1}}$	$-\sigma^2_{T_{r=r_2}}$	$-\sigma_{T_{r=r_1}}\times\sigma_{P_{r=r_1}}$	$-\sigma_{T_{r=r_2}}\times\sigma_{P_{r=r_2}}$	$\sigma^2_{P_{r=r_1}}$	$-\sigma^2_{P_{r=r_2}}$	$\sigma_{T_{r=r_2}}-\sigma_{T_{r=r_1}}$	$\sigma_{P_{r=r_1}}-\sigma_{P_{r=r_2}}$	
	$(psi)^2$	$(psi)^2$	$(psi)^2$	$(psi)^2$	$(psi)^2$	$(psi)^2$	(psi)	(psi)	(psi)
	$(11)^2 \times 10^{-6}$	$(14)^2 \times 10^{-6}$	$(11)\times(20) \times 10^{-6}$	$(14)\times(17) \times 10^{-6}$	$(20)^2 \times 10^{-6}$	$(17)^2 \times 10^{-6}$	$(11)+(14)$	$(20)-(17)$	$(27)+(28) \times 10^{-6}$
0.02	37.25	33.18	179.7	161.3	866.7	784.0	11,860	1440	.01330
0.03	79.48	78.50	176.8	162.8	393.2	337.8	17,780	1450	.01923
0.04	144.5	136.9	181.3	159.5	227.4	185.8	23,720	1450	.02517
0.05	226.5	212.6	183.6	156.7	148.8	115.6	29,630	1450	.03108
0.06	332.7	301.0	187.9	153.5	106.1	78.32	35,590	1450	.03704
0.08	595.8	529.9	193.1	148.7	62.57	41.73	47,430	1450	.04888
0.10	943.1	816.8	199.3	144.0	42.12	25.40	59,290	1450	.06074
0.12	1377	1158.	205.2	138.8	30.58	16.65	71,140	1450	.07259
0.14	1898	1556	211.3	133.7	23.52	11.49	83,010	1460	.08447

IN	30	31	32	33	34	35	36	37
	$\sigma_{L_{r=r_1 \& r_2}}$	$\sigma_{T_{r=r_2}}+\sigma_{P_{r=r_2}}$	$\sigma_{T_{r=r_2}}+\sigma_{L}$					σ_{TOTAL}
	(psi)	(psi)	(psi)	$(psi)^2$	$(psi)^2$	$(psi)^2$	$(psi)^2$	(psi)
	$\frac{\sum (21) \text{ to } (26)}{(29)}$	$(14)+(17)$	$(14)+(30)$	$(31)^2 \times 10^{-6}$	$(32)^2 \times 10^{-6}$	$(31)\times(32) \times 10^{-6}$	$(33)+(34)-(35) \times 10^{-6}$	$(36)^{1/2}$
0.02	-19,120	33760	-13360	1140	178.5	-451.0	1770	42,100
0.03	-14,730	27,240	-5870	742.0	34.46	-159.9	936.4	30,600
0.04	-11,590	25330	+ 110	641.6	.0121	+2.786	638.8	25,270
0.05	-9,430	25330	5150	641.6	26.52	130.4	537.7	23,180
0.06	-7,610	26200	9740	686.4	94.87	255.2	526.1	22,930
0.08	-5,220	29480	17800	869.1	316.8	524.7	661.2	25,710
0.10	-3,300	33620	25280	1130	639.1	849.9	919.2	30,320
0.12	-1,530	38110	32500	1452	1056	1239	1269	35,630
0.14	+107	42830	39550	1834	1564	1694	1704	41280

NOTE: WATER PRESSURE MAY VARY CONSIDERABLY WITHOUT SUBSTANTIALLY AFFECTING THE RESULTS.

TABLE 2

FLOW AND PRESSURE REQUIREMENTS FOR VARIOUS ANNULUS HEIGHTS OF THE MACH 10 THROAT SECTION

(1)	Annulus Height	$r_3 - r_2$, in.	0.080	0.090	0.100	0.110	0.120	0.130
(2)	r_3	(1) + 1, in.	1.080	1.090	1.100	1.110	1.120	1.130
(3)	$r_3 + r_2$	(2) + 1, in.	2.080	2.090	2.100	2.110	2.120	2.130
(4)	$(r_3 + r_2)^{0.8}$	$(3)^{0.8}$, $in.^{0.8}$	1.7965	1.8035	1.8110	1.8175	1.8240	1.8315
(5)		(1) x (4), $in.^{1.8}$	0.1437	0.1623	0.1811	0.1999	0.2189	0.2381
(6)	$X^{0.8}$	390 x (5), $\overline{gpm}^{0.8}$	56.07	63.30	70.62	77.98	85.40	92.86
(7)	H_2O Flow Required	$(6)^{1.25}$, gpm	153.6	178.5	204.0	231.5	259.5	288.5
(8)	$r_3^2 - r_2^2$	$(2)^2 - 1$, $in.^2$	0.1664	0.1881	0.2100	0.2321	0.2544	0.2769
(9)		0.1021/(8)	0.6136	0.5428	0.4862	0.4399	0.4013	0.3687
(10)	V	(7) x (9), fps	94.20	96.90	99.18	101.85	104.15	106.37
(11)	V^2	$(10)^2$, $(fps)^2$	8875	9390	9836	10380	10840	11320
(12)	$V^2/2g$	(11) /64.34, ft	137.9	146.0	152.9	161.4	168.6	176.0
(13)	Velocity Head	0.433 x (12), psi	59.75	63.22	66.23	69.90	72.98	76.20
(14)		(1) x (10)	7.535	8.718	9.918	11.21	12.49	13.83
(15)	Re x 10^{-3}	15 x (14)	113.1	130.8	148.8	168.1	187.4	207.4
(16)		$(15)^{0.32}$	4.540	4.757	4.957	5.160	5.336	5.512
(17)	f	0.0014 + 0.01371/(16)	0.00442	0.00428	0.00416	0.00406	0.00397	0.00389
(18)		0.01346 x (17) x 10^4	0.5949	0.5760	0.5598	0.5465	0.5344	0.5235
(19)		(11) x (18) / (1)	6.600	6.010	5.507	5.062	4.637	4.283
(20)	Total Head Required	4 x (19) + (13), psi	86.15	87.26	88.26	90.15	91.53	93.33

TABLE 3

CALCULATION OF HEAT TRANSFER COEFFICIENTS FOR THE MACH 10 THROAT SECTION

(Ref: Eqs. (21) – (14))

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		°R	°R	LB/IN.²	°R				°R		SLUGS/FT³			FT/SEC	FT	SLUGS/FT SEC				
STA	M	T_s	T_a	P_i		M^2	$M^2 T_s$		T'	RT'	ρ	$\sqrt{T_s}$	49.1 M	V	x	$\mu' \times 10^6$	ρV	ρVx	$RE \times 10^{-6}$	Log RE
					$\frac{(2)+(3)}{2}$	$(1)^2$	(2) × (6)	.03491×(7)	(5)+(8)	53.3×(9)	$\frac{4.476\times(4)}{(10)}$	$(2)^{1/2}$	49.1×(1)	(12)×(13)	$\frac{STA-16.3627}{12}$		(11)×(14)	(15)×(17)	$\frac{(18)}{(16)}$	
16.25	0.325	1895																		
17	0.400	1875																		
18	0.526	1834																		
19	0.682	1770																		
20	0.876	1677																		
20.571	1.000	1612	1060	1057	1336	1.000	1612	56.27	1392	74190	0.06377	40.15	49.10	1971	0.4340	0.741	126.8	55.03	74.26	7.8708
21	1.11	1552	1041	925.2	1296	1.232	1912	66.75	1363	72650	0.05700	39.40	54.50	2147	0.4698	0.731	122.4	57.50	78.66	7.8958
22	1.37	1407	990	655.4	1198	1.877	2641	92.20	1290	68760	0.04266	37.51	67.27	2523	0.5531	0.706	107.6	59.51	84.29	7.9258
23	1.63	1264	935	450.0	1100	2.657	3358	117.2	1217	64870	0.03105	35.56	80.03	2846	0.6364	0.680	88.37	56.24	82.70	7.9175
24	1.87	1139	875	312.6	1007	3.497	3983	139.0	1146	61080	0.02291	33.75	91.82	3099	0.7198	0.655	71.00	51.10	78.02	7.8922
25	2.11	1024	827	215.4	926	4.452	4559	159.2	1085	57830	0.01667	32.00	103.6	3315	0.8031	0.632	55.26	44.38	70.22	7.8465
26	2.32	932	782	155.0	857	5.382	5016	175.1	1032	55000	0.01261	30.53	113.9	3477	0.8864	0.611	43.84	38.86	63.60	7.8035
27	2.54	845	744	110.0	795	6.452	5452	190.3	985	52500	0.009378	29.07	124.7	3625	0.9698	0.592	34.00	32.97	55.69	7.7458
28	2.74	774	713	80.78	744	7.508	5811	202.9	947	50480	0.007163	27.82	134.5	3742	1.0531	0.577	26.80	28.22	48.91	7.6894
29	2.93	712	691	60.50	702	8.585	6112	213.4	915	48770	0.005553	26.68	143.9	3839	1.1364	0.564	21.32	24.23	42.96	7.6331
30	3.11	659	670	46.20	664	9.672	6374	222.5	887	47280	0.004374	25.67	152.7	3920	1.220	0.550	17.15	20.92	38.04	7.5802
32	3.43	577	649	28.98	613	11.765	6788	237.0	850	45300	0.002863	24.02	168.4	4045	1.386	0.536	11.43	15.84	29.55	7.4706
34	3.72	514	632	19.27	573	13.838	7113	248.3	821	43760	0.001971	22.67	182.6	4140	1.553	0.523	8.160	12.67	24.23	7.3844
36	4.00	461	618	13.17	540	16.000	7376	257.5	798	42530	0.001386	21.47	196.4	4217	1.720	0.513	5.845	10.05	19.59	7.2920
38	4.23	423	607	9.738	515	17.893	7569	264.2	779	41520	0.001050	20.57	207.7	4272	1.886	0.505	4.486	8.460	16.75	7.2240
40	4.42	394	595	7.640	495	19.536	7697	268.7	764	40720	0.000840	19.85	217.0	4307	2.053	0.498	3.618	7.428	14.92	7.1738
44	4.85	339	577	4.510	458	23.522	7974	278.4	736	39230	0.000515	18.41	238.1	4383	2.386	0.485	2.257	5.385	11.10	7.0453
48	5.23	299	560	2.902	430	27.353	8178	285.5	716	38160	0.000340	17.29	256.8	4440	2.720	0.464	1.510	4.107	8.851	6.9470

TABLE 3 (Concluded)

STA.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	8	9	0
								BTU/LB. °R						BTU/Sec Ft² °R						
STA.						T_s/T'	$C_f/2$	C_p'			P_R'	$P_R'^{2/3}$		h_a	r^*/r	A^*/A	h/h^*			
	20 − 2.5686	0.044 × 21	20 − 1.5000	23^3	$\frac{22}{24} \times 10^3$	$\frac{2}{9}$	25 × 26 × 10^3		27 × 28 × 10^3	29 × 17 × 10^4		$31^{2/3}$	$\frac{32.17}{32}$	30 × 33		35^2	$36^{0.875}$			
16.25														0.856	0.7263	0.5275	0.5714			
17														0.998	0.7929	0.6287	0.6662			
18														1.196	0.8792	0.7730	0.7983			
19														1.369	0.9499	0.9023	0.9140			
20														1.480	0.9932	0.9864	0.9881			
20.571	5.5022	0.24210	6.3708	258.57	0.9363	1.1580	1.0842	0.2639	0.2861	362.8	0.688	0.779	41.30	1.498	1.000	1.000	1.000			
21	5.5272	0.24320	6.3958	261.63	0.9296	1.1387	1.0585	0.2627	0.2781	340.4	0.687	0.778	41.35	1.408						
22	5.5572	0.24452	6.4258	265.33	0.9216	1.0907	1.0052	0.2599	0.2613	281.2	0.686	0.778	41.35	1.163						
23	5.5489	0.24415	6.4175	264.30	0.9238	1.0386	0.9595	0.2565	0.2461	217.5	0.683	0.776	41.46	0.9018						
24	5.5236	0.24304	6.3922	261.19	0.9305	0.9939	0.9248	0.2546	0.2355	167.2	0.681	0.774	41.56	0.6949						
25	5.4779	0.24103	6.3465	255.62	0.9429	0.9438	0.8899	0.2523	0.2245	124.1	0.680	0.773	41.62	0.5165						
26	5.4349	0.23914	6.3035	250.46	0.9548	0.9031	0.8623	0.2510	0.2164	94.87	0.680	0.773	41.62	0.3948						
27	5.3772	0.23660	6.2458	243.65	0.9711	0.8579	0.8331	0.2489	0.2074	70.52	0.680	0.773	41.62	0.2935						
28	5.3208	0.23412	6.1894	237.11	0.9874	0.8173	0.8070	0.2478	0.2000	53.60	0.680	0.773	41.62	0.2231						
29	5.2645	0.23164	6.1331	230.70	1.0041	0.7781	0.7813	0.2468	0.1928	41.10	0.680	0.773	41.62	0.1710						
30	5.2116	0.22931	6.0802	224.78	1.0202	0.7430	0.7580	0.2464	0.1868	32.04	0.680	0.773	41.62	0.1334						
32	5.1020	0.22449	5.9706	212.84	1.0547	0.6788	0.7159	0.2450	0.1754	20.05	0.681	0.774	41.56	0.0833						
34	5.0158	0.22070	5.8844	203.75	1.0832	0.6261	0.6782	0.2442	0.1656	13.51	0.682	0.775	41.51	0.0561						
36	4.9234	0.21663	5.7920	194.31	1.1149	0.5777	0.6441	0.2437	0.1570	9.177	0.683	0.776	41.46	0.0380						
38	4.8554	0.21364	5.7240	187.54	1.1392	0.5430	0.6186	0.2433	0.1505	6.751	0.685	0.777	41.40	0.0280						
40	4.8052	0.21143	5.6738	182.65	1.1576	0.5157	0.5970	0.2430	0.1451	5.250	0.686	0.778	41.35	0.0217						
44	4.6767	0.20577	5.5453	170.52	1.2067	0.4606	0.5558	0.2424	0.1347	3.040	0.688	0.779	41.30	0.0126						
48	4.5784	0.20145	5.4470	161.61	1.2465	0.4176	0.5205	0.2420	0.1260	1.903	0.689	0.780	41.24	0.00785						

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TABLE 4

SOLUTION OF HEAT BALANCE AND PRESSURE DROP EQUATIONS FOR THE MACH 10 THROAT SECTION

(Ref: Eqs. (10) – (16))

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	IN.	IN.	IN.	$IN.^2$	$IN.^2$	IN.	IN.	IN.	$IN.^{0.8}$	IN.	$IN.^2$	$IN.^2$	$IN.^2$	$IN.^2$	$IN.^2$	$IN.^{1.8}$	$\frac{BTU}{SEC\ FT^2\ °F}$	°F	$\frac{BTU}{SEC\ °F}$	$\frac{BTU}{SEC}$
STA.	r_1	r_2	r_3	r_2^2	r_3^2	r_1+r_2	r_2-r_1	r_3+r_2	$(r_3+r_2)^{0.8}$	r_3-r_2	$r_3^2-r_2^2$	A_1	A_m	A_2	A_3		h_a	T_r	$h_a A_1$	$h_a A_1 T_r$
				$(2)^2$	$(3)^2$	(1)+(2)	(2)−(1)	(3)+(2)	$(8)^{0.8}$	(3)−(2)	(5)−(4)	2π×(1)	π×(6)	2π×(2)	π×(11)	(9)×(10)			$\frac{(12)\times(17)}{144}$	(18)×(19)
16.25	1.2020	1.3760	1.5010	1.8934	2.2530	2.5780	0.1740	2.8770	2.3722	0.1250	0.3596	7.5524	8.0990	8.6456	1.1297	0.2965	0.856	1471	0.04489	66.03
17	1.1010	1.2610	1.3860	1.5901	1.9210	2.3620	0.1600	2.6470	2.1787	0.1250	0.3309	6.9178	7.4204	7.9231	1.0396	0.2723	0.998	1468	0.04794	70.38
18	0.9930	1.1370	1.2620	1.2928	1.5926	2.1300	0.1440	2.3990	2.0141	0.1250	0.2998	6.2392	6.6916	7.1440	0.9418	0.2518	1.196	1464	0.05182	75.86
19	0.9190	1.0520	1.1770	1.1067	1.3853	1.9710	0.1330	2.2290	1.8988	0.1250	0.2786	5.7742	6.1921	6.6099	0.8752	0.2374	1.369	1457	0.05489	79.97
20	0.8790	1.0070	1.1320	1.0140	1.2814	1.8860	0.1280	2.1390	1.8372	0.1250	0.2674	5.5229	5.9250	6.3272	0.8401	0.2296	1.480	1447	0.05676	82.13
20.571	0.8730	0.9990	1.1250	0.9980	1.2656	1.8720	0.1260	2.1240	1.8369	0.1260	0.2676	5.4852	5.8810	6.2769	0.8407	0.2314	1.498	1439	0.05706	82.11
21	0.8760	1.0040	1.1290	1.0080	1.2746	1.8800	0.1280	2.1330	1.8431	0.1250	0.2666	5.5041	5.9062	6.3083	0.8375	0.2304	1.408	1433	0.05382	77.12
22	0.9110	1.0430	1.1680	1.0878	1.3642	1.9540	0.1320	2.2110	1.8866	0.1250	0.2764	5.7240	6.1387	6.5534	0.8683	0.2358	1.163	1417	0.04623	65.51
23	0.9780	1.1200	1.2450	1.2544	1.5500	2.0980	0.1420	2.3650	1.9910	0.1250	0.2956	6.1450	6.5910	7.0372	0.9286	0.2489	0.9018	1401	0.03848	53.91
24	1.0710	1.2260	1.3510	1.5031	1.8252	2.2970	0.1550	2.5770	2.1325	0.1250	0.3221	6.7293	7.2162	7.7032	1.0119	0.2666	0.6949	1387	0.03247	45.04
25	1.1840	1.3550	1.4800	1.8360	2.1904	2.5390	0.1710	2.8350	2.3017	0.1250	0.3544	7.4393	7.9765	8.5137	1.1134	0.2877		1375		
26	1.3100	1.5000	1.6260	2.2500	2.6439	2.8100	0.1900	3.1260	2.4888	0.1260	0.3939	8.2310	8.8278	9.4248	1.2375	0.3136		1365		
27	1.4470	1.6570	1.7820	2.7456	3.1755	3.1040	0.2100	3.4390	2.6862	0.1250	0.4299	9.0918	9.7515	10.4112	1.3506	0.3358		1355		
28	1.5910	1.8220	1.9470	3.3197	3.7908	3.4130	0.2310	3.7690	2.8905	0.1250	0.4711	9.9965	10.7222	11.4480	1.4800	0.3613		1347		
29	1.7400	1.9920	2.1180	3.9681	4.4859	3.7320	0.2520	4.1100	3.0980	0.1260	0.5178	10.9327	11.7244	12.5161	1.6267	0.3903		1340		
30	1.8930	2.1670	2.2920	4.6959	5.2533	4.0600	0.2740	4.4590	3.3067	0.1250	0.5574	11.8940	12.7458	13.6156	1.7511	0.4133	0.1534	1335	0.01102	14.71
32	2.2050	2.5250	2.6500	6.3756	7.0225	4.7300	0.3200	5.1750	3.7252	0.1250	0.6469	13.8544	14.8597	15.8650	2.0323	0.4656		1326		
34	2.5220	2.8880	3.0130	8.3405	9.0782	5.4100	0.3660	5.9010	4.1375	0.1250	0.7377	15.8462	16.9960	18.1458	2.3176	0.5172		1319		
36	2.8408	3.2525	3.3775	10.5788	11.4075	6.0933	0.4117	6.6300	4.5419	0.1250	0.8287	17.8492	19.1426	20.4360	2.6034	0.5677		1313		
38	3.1599	3.6179	3.7430	13.0892	14.0100	6.7778	0.4580	7.3609	4.9379	0.1251	0.9208	19.8542	21.2931	22.7319	2.8928	0.6177		1309		
40	3.4791	3.9852	4.1084	15.8659	16.8790	7.4623	0.5041	8.0916	5.3268	0.1252	1.0131	21.8598	23.4435	25.0272	3.1827	0.6669		1305		
44	4.1174	4.7140	4.8393	22.2218	23.4188	8.8314	0.5966	9.5533	6.0831	0.1253	1.1970	25.8704	27.7446	29.6189	3.7605	0.7622		1299		
48	4.7556	5.4447	5.5702	29.6448	31.0271	10.2003	0.6891	11.0149	6.8168	0.1255	1.3823	29.8803	32.0452	34.2100	4.3426	0.8555	0.00785	1295	0.00163	2.109

TABLE 4 (Continued)

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	IN.	°F²	°F	°F					BTU / HR FT² °F	BTU / SEC. °F	BTU / SEC	°F	°F²			°F	°F²	°F	°F	°F
STA.	Am/t				$G^{.8}/D^{.2}$	$K^{2/3}/\nu^{1/3}$	$\mu_w^{0.14}$		h_w	$h_w A_e$	$h_w A_e T_b$									
	(13) / (7)	(20) × 1,784,000 / (21) × 10⁻³	(19) × 1,784,000 / (21)	(23) + 5535	393,525 / (16) × 10³			(26) × 1.023 / (27) × 10³	(25) × (28)	(14) × (29) / 518,400	(30) × (34)	(20) + (31) / (19)	(32)² × 10⁻³	(30) / (19)	(34)²	(32) × (34)	(32) × (24) × 10⁻³	(24) × (34)	2(36) + (38)	(39) + 5535
16.25	46.33	2542	1728	7263	1327	0.3494	0.9062	8.868	11770	0.1963	11.72	1732	3000	4.373	19.12	7574	12580	31760	46910	52440
17		2710	1846	7381	1445	0.3511	0.9023	8.950	12930	0.1976	11.97	1718	2951	4.122	16.99	7082	12680	30420	45580	50120
18		2921	1995	7530	1563	0.3534	0.8969	9.062	14160	0.1952	12.06	1697	2879	3.767	14.19	6393	12780	28370	41160	46690
19		3079	2114	7649	1658	0.3556	0.8914	9.175	15210	0.1939	12.22	1680	2821	3.532	12.48	5934	12850	27020	38880	44420
20		3162	2186	7721	1714	0.3581	0.8879	9.276	15900	0.1941	12.48	1667	2778	3.420	11.69	5701	12870	26400	37800	43340
20.574		3162	2197	7732	1701	0.3594	0.8879	9.310	15840	0.1918	12.47	1658	2749	3.361	11.30	5573	12820	25990	37140	42670
21		2970	2072	7607	1708	0.3603	0.8928	9.282	15850	0.1929	12.63	1668	2782	3.584	12.84	5978	12690	27260	39220	44750
22		2523	1780	7315	1669	0.3626	0.9036	9.230	15400	0.1947	12.99	1698	2883	4.212	17.74	7152	12420	30810	45110	50650
23		2076	1482	7017	1581	0.3646	0.9208	9.107	14400	0.1955	13.25	1745	3045	5.080	25.81	8865	12240	35650	53380	58920
24		1734	1250	6785	1476	0.3663	0.9352	9.009	13300	0.1976	13.58	1805	3258	6.086	37.04	10980	12250	41290	63250	68780
25																				
26																				
27																				
28																				
29																				
30		566	424	5959	952.2	0.3762	1.0271	8.424	8021	0.2107	15.59	2750	7562	19.12	365.6	52580	16,390	113,900	219100	224600
32																				
34																				
36																				
38																				
40																				
44																				
48	46.33	81	63	5598	460.0	0.3856	1.0957	8.094	3723	0.2457	19.41	13210	174500	150.8	22750	1,993,000	73,960	844,400	1,830,000	4,836,000

TABLE 4 (Continued)

	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	°F	°F²	°F²	°F²	°F²	°F	°F	°F	°F	°F	°F	BTU/SEC	°F/IN.	°F	FT/SEC	FT²/SEC²	SEC/FT			
STA.							T_w		T_a	$T_a - T_w$	$T_r - T_a$	q	ΔT_b	T_b	V	V^2		RE		
	$\frac{(40)}{(35)-1}$	$(41)^2 \times 10^{-3}$	$\{(33)+(37)-(22)\}10^{-3}$	$\frac{4\times(43)\times 10^{3}}{(35)-1}$	$(42)-(44)$ $\times 10^{-3}$	$(45)^{1/2}$	$\frac{(41)-(46)}{2}$	(34)×(47)	(32)−(48)	(49)−(47)	(18)−(49)	(19)×(51)	$\frac{(52)}{38.23}$		$\frac{88.22}{(15)}$	$(55)^2$	15000×(10)	(55)×(57)	$\left\{\frac{(58)}{1000}\right\}^{0.32}$	$\frac{0.01371}{(59)}$
														58.6						
16.25	2894	8375	13038	2878	5497	2345	274	1200	532	258	939	42.15	1.102	59.7	78.09	6098	1875	146,400	4.93	0.00278
17	3134	9824	12920	3232	6592	2568	283	1166	552	269	916	43.91	1.149	60.6	84.86	7201	1875	159,100	5.06	0.00271
18	3540	12530	12740	3863	8667	2943	298	1124	573	275	891	46.17	1.208	61.8	93.67	8774	1875	175,600	5.23	0.00262
19	3870	14970	12590	4387	10580	3253	308	1090	590	282	867	47.59	1.245	63.0	100.8	10161	1875	189,000	5.35	0.00256
20	4054	16430	12490	4672	11760	3428	313	1070	597	284	850	48.25	1.262	64.3	105.0	11025	1875	196,900	5.42	0.00253
20.571	4143	17160	12407	4818	12340	3513	315	1059	599	284	840	47.93	1.254	65.0	104.9	11004	1890	198,300	5.43	0.00252
21	3780	14290	12500	4224	10070	3173	303	1086	582	279	851	45.80	1.198	65.5	105.3	11088	1875	197,400	5.42	0.00253
22	3026	9157	12784	3054	6103	2471	278	1171	527	249	890	41.14	1.076	66.7	101.6	10322	1875	190,500	5.36	0.00256
23	2375	5641	13210	2130	3511	1874	250	1270	475	225	926	35.63	0.932	67.8	95.00	9025	1875	178,100	5.25	0.00261
24	1909	3644	13770	1529	2115	1454	228	1388	417	189	970	31.50	0.824	68.7	87.18	7600	1875	163,500	5.11	0.00268
25														69.6	79.23	6277	1875	148,600	4.95	0.00277
26															71.29	5082	1890	134,700	4.80	0.00286
27															65.32	4267	1875	122,500	4.66	0.00294
28															59.61	3553	1875	111,800	4.52	0.00303
29															54.23	2941	1890	102,500	4.40	0.00312
30	616.0	379.5	23390	256.6	122.9	350.6	133	2537	213	80	1122	12.36	0.323	74.0	50.38	2538	1875	94,460	4.29	0.00320
32															43.41	1884	1875	81,390	4.09	0.00335
34															38.06	1448	1875	71,360	3.92	0.00350
36															33.89	1148	1875	63,540	3.77	0.00364
38															30.50	930	1876	57,220	3.65	0.00376
40															27.72	768	1878	52,060	3.54	0.00387
44															23.46	550	1880	44,100	3.36	0.00408
48	212.5	45.17	248400	43.67	1.497	38.69	87	13110	100	13	1195	1.9	0.05	79.0	20.32	413	1882	38,240	3.21	0.00427

TABLE 2

FLOW AND PRESSURE REQUIREMENTS FOR VARIOUS ANNULUS HEIGHTS OF THE MACH 10 THROAT SECTION

1	Annulus Height	$r_3 - r_2$, in.	0.080	0.090	0.100	0.110	0.120	0.130
2	r_3	1 + 1, in.	1.080	1.090	1.100	1.110	1.120	1.130
3	$r_3 + r_2$	2 + 1, in.	2.080	2.090	2.100	2.110	2.120	2.130
4	$(r_3 + r_2)^{0.8}$	$3^{0.8}$, in.$^{0.8}$	1.7965	1.8035	1.8110	1.8175	1.8240	1.8315
5		1 x 4, in.$^{1.8}$	0.1437	0.1623	0.1811	0.1999	0.2189	0.2381
6	$X^{0.8}$	390 x 5, $\overline{\mathrm{gpm}}^{0.8}$	56.07	63.30	70.62	77.98	85.40	92.86
7	H_2O Flow Required	$6^{1.25}$, gpm	153.6	178.5	204.0	231.5	259.5	288.5
8	$r_3^2 - r_2^2$	$2^2 - 1$, in.^{2}	0.1664	0.1881	0.2100	0.2321	0.2544	0.2769
9		0.1021/8	0.6136	0.5428	0.4862	0.4399	0.4013	0.3687
10	V	7 x 9, fps	94.20	96.90	99.18	101.85	104.15	106.37
11	V^2	10^2, (fps)^2	8875	9390	9836	10380	10840	11320
12	$V^2/2g$	11/64.34, ft	137.9	146.0	152.9	161.4	168.6	176.0
13	Velocity Head	0.433 x 12, psi	59.75	63.22	66.23	69.90	72.98	76.20
14		1 x 10	7.535	8.718	9.918	11.21	12.49	13.83
15	Re x 10^{-3}	15 x 14	113.1	130.8	148.8	168.1	187.4	207.4
16		$15^{0.32}$	4.540	4.757	4.957	5.160	5.336	5.512
17	f	0.0014 + 0.01371/16	0.00442	0.00428	0.00416	0.00406	0.00397	0.00389
18		0.01346 x 17 x 10^4	0.5949	0.5760	0.5598	0.5465	0.5344	0.5235
19		11 x 18 / 1	6.600	6.010	5.507	5.062	4.637	4.283
20	Total Head Required	4 x 19 + 13, psi	86.15	87.26	88.26	90.15	91.53	93.33

TABLE 4 (Concluded)

	61	62	63	64	65	66			
STA.	f		Δp/IN.						
	.0014 + (60)	.01346 × (61) × 10^4	$\frac{(56) \times (62)}{(10)}$	$\frac{\Delta p_n + \Delta p_{n+1}}{2}$	(64) × Δℓ	ΣΔp			
16.25	0.00418	0.5626	2.7						
17	0.00411	0.5532	3.2	3.0	2.2	2.2			
18	0.00402	0.5411	3.8	3.5	3.5	5.7			
19	0.00396	0.5330	4.3	4.0	4.0	9.7			
20	0.00393	0.5290	4.7	4.5	4.5	14.2			
20.571	0.00392	0.5276	4.6	4.6	2.6	16.8			
21	0.00393	0.5290	4.7	4.6	2.0	18.8			
22	0.00396	0.5330	4.4	4.6	4.6	23.4			
23	0.00401	0.5397	3.9	4.2	4.2	27.6			
24	0.00408	0.5492	3.3	3.6	3.6	31.2			
25	0.00417	0.5613	2.8	3.0	3.0	34.2			
26	0.00426	0.5734	2.3	2.6	2.6	36.8			
27	0.00434	0.5842	2.0	2.2	2.2	39.0			
28	0.00443	0.5963	1.7	1.8	1.8	40.8			
29	0.00452	0.6084	1.4	1.6	1.6	42.4			
30	0.00460	0.6192	1.2	1.3	1.3	43.7			
32	0.00475	0.6394	1.0	1.1	2.2	45.9			
34	0.00490	0.6595	0.8	0.9	1.8	47.7			
36	0.00504	0.6784	0.6	0.7	1.4	49.1			
38	0.00516	0.6945	0.5	0.6	1.2	50.3			
40	0.00527	0.7093	0.4	0.4	0.8	51.1			
44	0.00548	0.7376	0.3	0.4	1.6	52.7			
48	0.00567	0.7632	0.2	0.2	0.8	53.5			

TABLE 5

CALCULATION OF OPTIMUM THROAT THICKNESS OF THE MACH 12 THROAT SECTION

(Ref: Eqs. (1) – (9))

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
															OUTSIDE PRES. STRESS			INSIDE PRES. STRESS		
t	ΔT	K	K^2	K^2+1	K^2-1	$\ln K$				$-\bar{\gamma}_a$	$-\sigma_{r_{r=r_1}}$		$\bar{\gamma}_b$	$\sigma_{r_{r=r_2}}$	(AIR)	(WATER)	(COMB.)	(AIR)	(WATER)	(COMB.)
IN.	°F									LB/IN.² °F	LB/IN.²		LB/IN.² °F		LB/IN.²	LB/IN.²	LB/IN.²	LB/IN.²	LB/IN.²	LB/IN.²
	$4600 \times t$	$\frac{r_1+t}{r_1}$	$(2)^2$	$(3)+1$	$(3)-1$		$\frac{2\times(6)}{(5)}$	$(3)\times(7)$	$-(1-(8))$	$\frac{122.5\cdot(9)}{(6)}$	$(1)\times(10)$	$1-(7)$	$\frac{122.5\cdot(12)}{(6)}$	$(1)\times(13)$	$\frac{2537}{(5)}$	$\frac{500\cdot(4)}{(5)}$	$(15)-(16)$	$\frac{1268\cdot(4)}{(5)}$	$\frac{1000}{(5)}$	$(18)-(19)$
0.015	69	1.0268	1.0543	2.0543	0.0543	0.02645	0.9742	1.0271	0.0271	125.5	8660	0.0257	119.0	8210	46720	18920	27800	47970	18420	29550
0.020	92	1.0357	1.0727	2.0727	0.0727	0.03508	0.9651	1.0353	0.0353	123.3	11340	0.0349	121.9	11220	34900	14200	20640	36150	13760	22390
0.025	115	1.0446	1.0912	2.0912	0.0912	0.04363	0.9568	1.0441	0.0441	123.8	14240	0.0432	121.3	13950	27820	11460	16360	29080	10960	18120
0.030	138	1.0536	1.1101	2.1101	0.1101	0.05221	0.9484	1.0528	0.0528	123.9	17100	0.0516	121.1	16710	23040	9580	13460	24300	9080	15220
0.040	184	1.0714	1.1479	2.1479	0.1479	0.06897	0.9327	1.0706	0.0706	125.4	23080	0.0673	119.5	21990	17150	7260	9890	18410	6760	11650
0.060	276	1.1071	1.2258	2.2258	0.2258	0.10174	0.9012	1.1047	0.1047	126.1	34800	0.0988	119.0	32850	11240	4925	6315	12500	4430	8070
0.080	368	1.1428	1.3061	2.3061	0.3061	0.13348	0.8721	1.1390	0.1390	127.6	46960	0.1279	117.4	43200	8290	3770	4520	9550	3270	6280
0.100	460	1.1786	1.3890	2.3890	0.3890	0.16433	0.8449	1.1736	0.1736	129.4	59520	0.1551	115.6	53180	6520	3070	3450	7790	2570	5220
0.120	552	1.2143	1.4745	2.4745	0.4745	0.19417	0.8184	1.2067	0.2067	130.4	71980	0.1816	114.6	63260	5350	2610	2740	6610	2110	4500

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
	$\sigma_{r_{r=r_1}}^2$	$\sigma_{r_{r=r_2}}^2$	$-\sigma_{r_{r=r_1}}\times\sigma_{\theta_{r=r_1}}$	$-\sigma_{r_{r=r_2}}\times\sigma_{\theta_{r=r_2}}$	$\sigma_{\theta_{r=r_1}}^2$	$-\sigma_{\theta_{r=r_2}}^2$	$\sigma_{r_{r=r_2}}-\sigma_{r_{r=r_1}}$	$\sigma_{\theta_{r=r_1}}-\sigma_{\theta_{r=r_2}}$	$\Sigma (27),(28)$	$\sigma_{L_{r=r_1,r_2}}$	$\sigma_{r_{r=r_2}}+\sigma_{\theta_{r=r_2}}$	$\sigma_{r_{r=r_2}}+\sigma_L$					σ_{TOTAL}
	LB²/IN.⁴	LB²/IN.⁴	LB²/IN.⁴	LB²/IN.⁴	LB²/IN.⁴	LB²/IN.⁴	LB/IN.²	LB/IN.²	LB/IN.²	LB/IN.²	LB/IN.²	LB/IN.²	LB²/IN.⁴	LB²/IN.⁴	LB²/IN.⁴	LB²/IN.⁴	LB/IN.²
	$(11)^2\times10^{-6}$	$(14)^2\times10^{-6}$	$(11)\times(20)$ $\times10^{-6}$	$(14)\times(17)$ $\times10^{-6}$	$(20)^2\times10^{-6}$	$(17)^2\times10^{-6}$	$(11)+(14)$	$(20)-(17)$	$(27)+(28)$ $\times10^{-6}$	$\frac{\Sigma(21)\text{ to }(26)}{(29)}$	$(14)+(17)$	$(14)+(30)$	$(31)^2\times10^{-6}$	$(32)^2\times10^{-6}$	$(31)\times(32)$ $\times10^{-6}$	$(33)+(34)$ $-(35)\times10^{-6}$	$(36)^{1/2}$
0.015	75.0	67.4	255.9	228.2	873.2	772.8	16870	1750	0.01862	-20203	36010	-11990	1296.7	143.8	-431.9	1872.4	48270
0.020	128.6	125.9	253.9	231.6	501.3	426.0	22560	1750	0.02431	-16760	31860	-5540	1015.1	30.7	-176.5	1222.3	34960
0.025	202.8	194.6	258.0	228.2	328.3	267.6	28190	1760	0.02995	-13940	30310	+ 14	918.7	0.0	0.4	918.3	30300
0.030	292.4	279.2	260.3	224.9	231.6	181.2	33810	1760	0.03557	-11850	30170	4860	910.2	23.6	146.6	787.2	28060
0.040	532.7	483.6	268.9	217.5	135.7	97.8	45070	1760	0.04683	-8530	31880	13460	1016.3	181.2	429.2	768.3	27720
0.060	1211.0	1079.1	280.8	207.4	65.1	39.9	67650	1755	0.06940	-4770	39160	28080	1533.9	788.4	1099.7	1222.6	34970
0.080	2205.2	1866.2	294.9	195.3	39.4	20.4	90160	1760	0.09192	-1440	47720	41760	2277.2	1744.1	1992.9	2028.4	45040
0.100	3542.6	2828.1	310.7	183.5	27.2	11.9	112700	1770	0.11447	+2060	56630	55240	3207.0	3051.3	3128.2	3130.1	55950
0.120	5181.1	4001.8	323.9	173.3	20.2	7.5	135240	1760	0.13700	+5070	66000	68330	4356.0	4669.1	4509.8	4515.3	67200

NOTE: WATER PRESSURE MAY VARY CONSIDERABLY WITHOUT SUBSTANTIALLY AFFECTING THE RESULTS.

TABLE 6

FLOW AND PRESSURE REQUIREMENTS FOR VARIOUS ANNULUS HEIGHTS OF THE MACH 12 THROAT SECTION

1	Annulus Height	$r_3 - r_2$, in.	0.100	0.110	0.120	0.130	0.140	0.150
2	r_3	1 + 0.660, in.	0.760	0.770	0.780	0.790	0.800	0.810
3	$r_3 + r_2$	2 + 0.660, in.	1.420	1.430	1.440	1.450	1.460	1.470
4	$(r_3 + r_2)^{0.8}$	$3^{0.8}$, in.$^{0.8}$	1.3238	1.3313	1.3387	1.3462	1.3536	1.3610
5		1 x 4, in.$^{1.8}$	0.1324	0.1464	0.1606	0.1750	0.1895	0.2042
6	$X^{0.8}$	1107 x 5, $\overline{gpm}^{0.8}$	146.6	162.1	177.8	193.8	209.7	226.0
7	H_2O Flow Required	$6^{1.25}$, gpm	512	577	648	723	798	875
8	$r_3^2 - r_2^2$	2^2 - 0.4356, in.^{2}	0.1420	0.1573	0.1728	0.1885	0.2044	0.2205
9		0.1021/8	0.7187	0.6492	0.5908	0.5416	0.4993	0.4629
10	V	7 x 9, fps	368.2	374.4	382.8	391.4	398.6	405.0
11	V^2	10^2, (fps)^2	135,500	140,200	146,600	153,200	158,800	164,000
12	$V^2/2g$	11 /64.34, ft	2106	2179	2278	2381	2468	2549
13	Velocity Head	0.433 x 12, psi	912.5	944.5	987.5	1031	1070	1105
14		1 x 10	36.82	41.20	45.96	50.89	55.80	60.77
15	Re x 10^{-3}	15 x 14	552.4	618.0	689.3	763.5	837.2	911.5
16		$15^{0.32}$	7.54	7.82	8.10	8.37	8.61	8.85
17	f	0.0014 + 0.01371/16	0.00322	0.00315	0.00309	0.00304	0.00299	0.00295
18		0.01346 x 17 x 10^4	0.4333	0.4238	0.4158	0.4092	0.4023	0.3970
19		11 x 18 /1	58.73	54.02	50.78	48.22	45.64	43.41
20	Total Head Required	2.5 x 19 + 13, psi	1059	1079	1115	1152	1184	1214

TABLE 7
CALCULATION OF HEAT TRANSFER COEFFICIENTS FOR THE MACH 12 THROAT SECTION
(Ref: Eqs. (21) – (24))

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		°R	°R	LB/IN.²	°R				°R		SLUGS/FT³			FT/SEC	FT	SLUGS/FT SEC				
STA.	M	T_s	T_a	p_1		M^2	$M^2 T_s$		T'	RT'	ρ	$\sqrt{T_s}$	49.1 M	V	x	$\nu' \times 10^6$	ρV	$\rho V x$	$RE \times 10^{-6}$	Log RE
					$\frac{(2)+(5)}{2}$	$(1)^2$	(2) × (6)	.03491·(7)	(5) + (8)	53.3·(9)	$\frac{4.476 \cdot (4)}{(10)}$	$(2)^{1/2}$	49.1·(1)	(12)·(13)	$\frac{STA. - 14.9582}{12}$		(11)·(14)	(15)·(17)	$\frac{(18)}{(16)}$	
14.0	0.17	2386																		
14.5	0.21	2379																		
15	0.26	2369																		
16	0.38	2332																		
17	0.55	2261																		
18	0.79	2134																		
18.7041	1.00	2000	1116	1268	1558	1.000	2000	69.82	1628	86770	0.06541	44.72	49.10	2196	0.3127	0.817	143.6	44.90	54.96	7.7400
19	1.12	1918	1092	1096	1505	1.254	2406	83.99	1589	84690	0.05792	43.80	54.99	2408	0.3373	0.804	139.5	47.05	58.52	7.7673
20	1.47	1676	1010	682.8	1343	2.161	3622	126.4	1469	78300	0.03903	40.94	72.18	2955	0.4206	0.767	115.3	48.50	63.23	7.7909
21	1.81	1450	936	411.4	1193	3.276	4750	165.8	1359	72430	0.02542	38.08	88.87	3384	0.5040	0.731	86.02	43.35	59.31	7.7731
22	2.14	1252	872	246.5	1062	4.580	5734	200.2	1262	67260	0.01640	35.38	105.1	3718	0.5873	0.697	60.98	35.81	51.38	7.7108
23	2.46	1086	816	149.5	951	6.052	6572	229.4	1180	62890	0.01064	32.95	120.8	3980	0.6706	0.667	42.35	28.40	42.58	7.6292
24	2.75	955	767	95.47	861	7.562	7222	252.1	1113	59320	0.007204	30.90	135.0	4172	0.7540	0.642	30.06	22.66	35.30	7.5478
25	3.01	853	723	64.37	788	9.060	7728	269.8	1058	56390	0.005109	29.21	147.8	4317	0.8373	0.622	22.06	18.47	29.69	7.4726
26	3.25	771	690	45.12	730	10.562	8144	284.3	1014	54050	0.003736	27.77	159.6	4432	0.9206	0.605	16.56	15.24	25.19	7.4012
28	3.69	645	640	24.10	642	13.616	8782	306.9	949	50580	0.002133	25.40	181.2	4602	1.0873	0.578	9.816	10.67	18.46	7.26662
30	4.09	552	602	14.03	577	16.728	9234	322.4	899	47920	0.001310	23.49	200.8	4717	1.2540	0.557	6.179	7.748	13.91	7.1333
32	4.46	482	580	8.719	531	19.892	9588	334.7	866	46160	0.0008454	21.95	219.0	4807	1.4206	0.543	4.064	5.773	10.63	7.0265
34	4.77	432	563	5.957	498	22.753	9829	343.1	841	44820	0.0005967	20.78	234.2	4867	1.5873	0.532	2.904	4.610	8.665	6.9378
36	5.06	392	550	4.231	471	25.604	10037	350.4	821	43760	0.0004328	19.80	248.4	4918	1.7540	0.523	2.128	3.732	7.136	6.8534
38	5.32	360	548	3.146	454	28.302	10189	355.7	810	43170	0.0003262	18.97	261.2	4955	1.9206	0.519	1.616	3.104	5.981	6.7768
40	5.56	334	546	2.417	440	30.914	10325	360.4	800	42640	0.0002537	18.28	273.0	4990	2.0873	0.514	1.266	2.642	5.140	6.7110
44	6.01	292	545	1.504	419	36.120	10547	368.2	787	41950	0.0001605	17.09	295.1	5043	2.4206	0.508	0.8094	1.959	3.856	6.5861
48	6.44	258	545	.9804	402	41.474	10700	373.5	775	41310	0.0001062	16.06	316.2	5078	2.7540	0.503	0.5394	1.486	2.954	6.4603

TABLE 7 (Concluded)

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	8	9	0
								$\frac{BTU}{LB\ °R}$						$\frac{BTU}{SEC\ FT^2\ °R}$						
STA.						T_s/T'	$C_f/2$	C_p'			PR	$PR^{2/3}$		h_a	r^*/r	A^*/A	h/h^*			
	(20) − 2.3686	0.044 · (21)	(20) − 1.5000	$(23)^3$	$\frac{(22) \times 10^3}{(24)}$	$\frac{(2)}{(9)}$	(25) × (26)		(27) × (28) × 10^4	(29) × (17) × 10^{-3}		$(31)^{2/3}$	$\frac{32.17}{(32)}$	(30) × (33) × 10^{-4}		$(35)^2$	$(36)^{.875}$			
14.0														0.6501	0.5418	0.2935	0.3420			
14.5														0.7652	0.5945	0.3534	0.4025			
15														0.8986	0.6516	0.4246	0.4727			
16														1.217	0.7749	0.6004	0.6400			
17														1.565	0.8948	0.8008	0.8234			
18														1.835	0.9799	0.9602	0.9651			
18.7041	5.3714	0.23634	6.2400	242.97	0.9727	1.228	1.194	0.2705	3230	463.8	0.696	0.785	40.98	1.901	1.000	1.000	1.000			
19	5.3987	0.23754	6.2673	246.17	0.9649	1.207	1.165	0.2692	3136	437.5	0.695	0.785	40.98	1.793						
20	5.4223	0.23858	6.2909	248.96	0.9583	1.141	1.093	0.2650	2896	333.9	0.690	0.781	41.19	1.375						
21	5.4045	0.23780	6.2731	246.86	0.9633	1.067	1.028	0.2611	2684	230.9	0.687	0.778	41.35	0.9548						
22	5.3422	0.23506	6.2108	239.58	0.9811	0.9921	0.9734	0.2576	2507	152.9	0.684	0.776	41.46	0.6339						
23	5.2606	0.23147	6.1292	230.26	1.005	0.9203	0.9249	0.2548	2357	99.82	0.682	0.775	41.51	0.4144						
24	5.1792	0.22788	6.0478	221.20	1.030	0.8580	0.8837	0.2525	2231	67.06	0.681	0.774	41.56	0.2787						
25	5.1040	0.22458	5.9726	213.05	1.054	0.8062	0.8497	0.2507	2130	46.99	0.680	0.773	41.62	0.1956						
26	5.0326	0.22143	5.9012	205.50	1.078	0.7604	0.8197	0.2491	2042	33.82	0.680	0.773	41.62	0.1408						
28	4.8976	0.21549	5.7662	191.72	1.124	0.6797	0.7640	0.2472	1889	18.54	0.680	0.773	41.62	0.0772						
30	4.7647	0.20965	5.6333	178.77	1.173	0.6140	0.7202	0.2460	1772	10.95	0.680	0.773	41.62	0.0466						
32	4.6579	0.20495	5.5265	168.79	1.214	0.5566	0.6757	0.2452	1657	6.734	0.682	0.775	41.51	0.0280						
34	4.5692	0.20104	5.4378	160.79	1.250	0.5137	0.6421	0.2445	1570	4.559	0.683	0.776	41.46	0.0189						
36	4.4848	0.19733	5.3534	153.42	1.286	0.4775	0.6141	0.2441	1499	3.190	0.684	0.776	41.46	0.0132						
38	4.4082	0.19396	5.2768	146.93	1.320	0.4444	0.5866	0.2438	1430	2.311	0.684	0.776	41.46	0.00958						
40	4.3424	0.19106	5.2110	141.50	1.350	0.4175	0.5636	0.2436	1373	1.738	0.685	0.777	41.40	0.00720						
44	4.2175	0.18557	5.0861	131.57	1.410	0.3710	0.5231	0.2433	1273	1.030	0.686	0.778	41.35	0.00426						
48	4.0917	0.18003	4.9603	122.05	1.475	0.3329	0.4910	0.2431	1194	0.6439	0.686	0.778	41.35	0.00266						

TABLE 8

SOLUTION OF HEAT BALANCE AND PRESSURE DROP EQUATIONS FOR THE MACH 12 THROAT SECTION

(Ref: Eqs. (10) – (16))

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	IN.	IN.	IN.	$IN.^2$	$IN.^2$	IN.	IN.	IN.	$IN.^{0.8}$	IN.	$IN.^2$	$IN.^2$	$IN.^2$	$IN.^2$	$IN.^2$	$IN.^{1.8}$	$\frac{BTU}{sec\ Ft^2\ ^\circ F}$	°F	$\frac{BTU}{SEC\ ^\circ F}$	$\frac{BTU}{SEC}$
STA.	r_1	r_2	r_3	r_2^2	r_3^2	$r_1 + r_2$	$r_2 - r_1$	$r_3 + r_2$	$(r_3+r_2)^{0.8}$	$r_3 - r_2$	$r_3^2 - r_2^2$	A_1	A_m	A_2	A_3		h_a	T_r	$h_a A_1$	$h_a A_1 T_r$
				$(2)^2$	$(3)^2$	(1) + (2)	(2) − (1)	(3) + (2)	$(8)^{0.8}$	(3) − (2)	(5) − (4)	$2\pi \cdot (1)$	$\pi \cdot (6)$	$2\pi \cdot (2)$	$\pi \cdot (11)$	(9) × (10)			$\frac{(12) \times (17)}{144}$	(18) × (19)
14.0	1.0336	1.2182	1.3824	1.4840	1.9110	2.2518	0.1846	2.6006	2.1489	0.1642	0.4270	6.4943	7.0742	7.6542	1.3414	0.3528	0.6501	1939	0.02932	56.85
14.5	0.9420	1.1102	1.2865	1.2325	1.6551	2.0522	0.1682	2.3967	2.0123	0.1763	0.4226	5.9188	6.4472	6.9756	1.3276	0.3548	0.7652	1938	0.03145	60.95
15	0.8594	1.0129	1.1906	1.0260	1.4175	1.8723	0.1535	2.2035	1.8814	0.1777	0.3915	5.3998	5.8820	6.3642	1.2299	0.3343	0.8986	1937	0.03370	65.28
16	0.7227	0.8518	1.0064	0.7256	1.0128	1.5745	0.1291	1.8582	1.6416	0.1546	0.2872	4.5408	4.9464	5.3520	0.9023	0.2538	1.217	1933	0.03838	74.19
17	0.6258	0.7376	0.8756	0.5440	0.7667	1.3634	0.1118	1.6132	1.4661	0.1380	0.2227	3.9320	4.2832	4.6345	0.6996	0.2023	1.565	1925	0.04273	82.26
18	0.5715	0.6736	0.8046	0.4537	0.6474	1.2451	0.1021	1.4782	1.3671	0.1310	0.1937	3.5908	3.9116	4.2324	0.6085	0.1791	1.835	1912	0.04576	87.49
18.7041	0.5600	0.6600	0.7900	0.4356	0.6241	1.2200	0.1000	1.4500	1.3462	0.1300	0.1885	3.5186	3.8327	4.1469	0.5922	0.1750	1.901	1898	0.04645	88.16
19	0.5620	0.6624	0.7925	0.4388	0.6280	1.2244	0.1004	1.4549	1.3498	0.1300	0.1892	3.5311	3.8466	4.1620	0.5944	0.1755	1.793	1890	0.04397	83.10
20	0.5980	0.7048	0.8366	0.4967	0.6999	1.3028	0.1068	1.5414	1.4136	0.132	0.2032	3.7573	4.0929	4.4284	0.6384	0.1866	1.375	1865	0.03588	66.92
21	0.6720	0.7920	0.9371	0.6273	0.8782	1.4640	0.1200	1.7291	1.5497	0.145	0.2509	4.2223	4.5993	4.9763	0.7882	0.2247	0.9548	1841	0.02800	51.55
22	0.7732	0.9113	1.0937	0.8305	1.1962	1.6845	0.1381	2.0050	1.7447	0.182	0.3657	4.8582	5.2920	5.7259	1.1489	0.3175	0.6339	1821	0.02139	38.95
23	0.8924	1.0518	1.2718	1.1063	1.6175	1.9442	0.1594	2.3236	1.9628	0.220	0.5112	5.6071	6.1079	6.6086	1.6060	0.4318	0.4144	1803	0.01614	29.10
24	1.0233	1.2060	1.4498	1.4544	2.1019	2.2293	0.1827	2.6558	2.1840	0.244	0.6475	6.4296	7.0035	7.5775	2.0342	0.5329	0.2787	1790	0.01244	22.27
25	1.1619	1.3694	1.6278	1.8752	2.6497	2.5313	0.2075	2.9972	2.4060	0.258	0.7745	7.3004	7.9523	8.6042	2.4332	0.6207	0.1956	1779		
26	1.3058	1.5390	1.8058	2.3685	3.2609	2.8448	0.2332	3.3448	2.6275	0.267	0.8924	8.2046	8.9372	9.6698	2.8036	0.7015	0.1408	1770		
28	1.6036	1.8900	2.1618	3.5721	4.6734	3.4936	0.2864	4.0518	3.0628	0.272	1.1013	10.076	10.975	11.875	3.4598	0.8331	0.0772	1757		
30	1.9094	2.2460	2.5179	5.0445	6.3398	4.1554	0.3366	4.7639	3.4863		1.2953	11.997	13.054	14.112	4.0693	0.9483	0.0466	1748	0.003882	6.78
32	2.2198	2.6021	2.8739	6.7709	8.2593	4.8219	0.3823	5.4760	3.8974		1.4884	13.947	15.148	16.349	4.6759	1.0601	0.0280	1740		
34	2.5331	2.9581	3.2300	8.7504	10.433	5.4912	0.4250	6.1881	4.2976		1.6826	15.916	17.251	18.586	5.2860	1.1689	0.0189	1735		
36	2.8483	3.3142	3.5860	10.984	12.859	6.1625	0.4659	6.9002	4.6891		1.911	17.865	19.360	20.824	6.004	1.2754	0.0132	1731		
38	3.1648	3.6702	3.9421	13.470	15.540	6.8350	0.5054	7.6123	5.0724		2.070	19.885	21.473	23.060	6.503	1.3797	0.0096	1728		
40	3.4821	4.0263	4.2981	16.211	18.474	7.5084	0.5442	8.3244	5.4475		2.263	21.879	23.588	25.298	7.109	1.4817	0.0072	1725		
44	4.1184	4.7384	5.0102	22.452	25.102	8.8568	0.6200	9.7486	6.1823		2.650	25.877	27.824	29.772	8.325	1.6816	0.0043	1721		
48	4.7558	5.4505	5.7223	29.708	32.745	10.2063	0.6947	11.1728	6.8950	0.272	3.037	29.882	32.064	34.246	9.541	1.8754	0.00266	1717	0.0000552	0.949

TABLE 8 (Continued)

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	IN.	$°F^2$	°F	°F					$\frac{BTU}{HR\ FT^2\ °F}$	$\frac{BTU}{SEC\ °F}$	$\frac{BTU}{SEC}$	°F	$°F^2$			°F	$°F^2$	°F	°F	°F
STA.	Am/t				$G^{0.8}/D^2$	$K^{2/3}/\mu^{1/3}$	$\mu_w^{0.14}$		h_w	$h_w A_z$	$h_w A_z T_b$									
	$\frac{(13)}{(7)}$	$\frac{(20) \times 1{,}784}{(21)} \times 10^6$	$\frac{(19) \times 1{,}784.002}{(21)} \times 10^6$	(23)+5535	$\frac{833{,}000}{(16)}$			$\frac{(26)}{(27)}$ 0.023	(25) × (28)	$\frac{(14) \times (29)}{518{,}400}$	(30) × (54)	$\frac{(20)+(31)}{(19)}$	$(32)^2$	$\frac{(30)}{(19)}$	$(34)^2$	(32) × (34)	(24) × (32) $\times 10^{-3}$	(24) × (34)	2×(36)+(38)	(39)+5535
14.0	38.33	2,646,000	1365	6900	2,361,000	0.3500	0.9600	0.008385	19,800	0.2923	17.54	2537	6437×10^3	9.469	99.39	25,290	17,500	68,790	119,400	124,900
14.5	↓	2,837,000	1464	6999	2,348,000	0.3508	0.9394	0.008589	20,170	0.2714	16.39	2459	6047 "	8.630	74.48	21,220	17,210	60,400	102,800	108,400
15		3,038,000	1568	7103	2,492,000	0.3517	0.9315	0.008684	21,640	0.2657	16.18	2417	5842 "	7.884	62.16	19,060	17,170	56,000	94,120	99,660
16		3,453,000	1786	7321	3,282,000	0.3526	0.9310	0.008711	28,590	0.2952	18.12	2405	5784 "	7.692	59.17	18,500	17,610	56,310	93,310	98,840
17		3,829,000	1989	7524	4,118,000	0.3538	0.9300	0.008750	36,030	0.3221	19.97	2392	5722 "	7.538	56.82	18,030	18,000	56,720	92,780	98,320
18		4,072,000	2130	7665	4,651,000	0.3549	0.9304	0.008773	40,800	0.3331	20.85	2368	5607 "	7.279	52.98	17,240	18,150	55,790	90,270	95,800
18.7041		4,103,000	2162	7697	4,760,000	0.3556	0.9297	0.008797	41,870	0.3349	21.10	2352	5532 "	7.210	51.98	16,960	18,100	55,500	89,420	94,960
19		3,868,000	2046	7581	4,746,000	0.3560	0.9346	0.008761	41,580	0.3338	21.10	2370	5617 "	7.592	57.64	17,990	17,970	57,550	93,530	99,060
20		3,115,000	1670	7205	4,464,000	0.3569	0.9522	0.008621	38,480	0.3287	20.94	2449	5998 "	9.161	83.92	22,440	17,640	66,000	110,900	116,400
21		2,399,000	1303	6838	3,707,000	0.3579	0.9676	0.008507	31,540	0.3028	19.44	2535	6426 "	10.81	116.9	27,410	17,330	73,950	128,800	134,300
22		1,813,000	996	6531	2,624,000	0.3586	0.9708	0.008496	22,290	0.2462	15.90	2564	6574 "	11.51	132.5	29,510	16,740	75,170	134,200	139,700
23	↓	1,354,000	751	6286	1,929,000	0.3592	0.9791	0.008438	16,280	0.2075	13.47	2638	6959 "	12.86	165.3	33,910	16,580	80,840	148,700	154,200
24	38.33	1,036,000	579	6114	1,563,000	0.3596	0.9936	0.008324	13,010	0.1902	12.38	2785	7756 "	15.29	233.8	42,580	17,030	93,480	178,600	184,200
25																				
26																				
28																				
30	38.78	312,000	178	5713	878,000	0.3607	1.0704	0.007750	6,804	0.1852	12.17	4882	23.83×10^6	47.71	2276	232,900	27,890	272,600	738,400	743,900
32																				
34																				
36																				
38																				
40																				
44																				
48	46.16	36,700	21.4	5556	444,000	0.3631	1.1277	0.007406	3,289	0.2173	14.56	28070	787.8×10^6	393.3	154,600	11.04×10^6	156.0×10^6	2.18×10^6	24.26×10^6	24.26×10^6

TABLE 8 (Continued)

	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	°F	°F²	°F²	°F²	°F²	°F	°F	°F	°F	°F	°F	BTU/SEC	°F	°F	FT/SEC	FT²/SEC²	SEC/FT			
STA							T_w		T_a	$T_a - T_w$	$T_r - T_a$	q	ΔT_b	T_b	V	V^2		RE		
	$\frac{(40)}{(35)-1}$	$(41)^2$	$(33)+(37)-(22)$	$\frac{4\times(43)}{(35)-1}$	$(42)-(44)$	$(45)^{1/2}$	$\frac{(41)-(46)}{2}$	$(34)\times(47)$	$(32)-(48)$	$(49)-(47)$	$(18)-(49)$	$(19)\times(51)$	$\frac{(52)}{97.35}$		$\frac{224.56}{(15)}$	$(55)^2$	$15000-(10)$	$(55)\times(57)$	$\left(\frac{(58)}{1000}\right)^{.32}$	$\frac{0.01371}{(59)}$
140	1269	1612×10³	21.29×10⁶	886×10³	746×10³	864	202	2019	518	316	1421	41.66	0.4280	60.0	167.4	28,023	2463	412,300	6.87	0.00200
14.5	1475	2176 "	20.42 "	1112 "	1064 "	1032	222	1912	547	325	1391	43.75	0.4494	60.4	169.1	28,608	2644	447,100	7.04	0.00195
15	1629	2654 "	19.97 "	1306 "	1348 "	1161	234	1845	572	338	1365	46.00	0.4725	60.9	182.6	33,335	2666	486800	7.24	0.00189
16	1699	2887 "	19.94 "	1371 "	1516 "	1231	234	1800	605	371	1328	50.97	0.5236	61.4	248.9	61,941	2319	577200	7.63	0.00180
17	1761	3101 "	19.89 "	1426 "	1675 "	1294	234	1760	632	398	1293	55.25	0.5675	62.0	321.0	103,030	2070	664500	8.00	0.00171
18	1843	3397 "	19.68 "	1515 "	1882 "	1372	235	1714	654	418	1258	57.57	0.5913	62.6	369.0	136,190	1965	725100	8.22	0.00167
18.7041	1862	3467 "	19.54 "	1532 "	1935 "	1391	235	1698	654	418	1244	57.78	0.5936	63.0	379.2	143,793	1950	739400	8.26	0.00166
19	1749	3059 "	19.72 "	1392 "	1667 "	1291	229	1738	632	403	1258	55.31	0.5682	63.2	377.8	142,725	1950	736700	8.25	0.00166
20	1404	1971 "	20.52 "	990 "	981 "	990	207	1896	553	346	1312	47.07	0.4836	63.7	351.8	123,728	1980	696600	8.11	0.00169
21	1159	1343 "	21.36 "	737 "	606 "	778	190	2059	476	286	1365	38.22	0.3926	64.2	284.9	81,168	2175	619600	7.81	0.00176
22	1062	1128 "	21.50 "	654 "	474 "	688	187	2152	412	225	1409	30.14	0.3096	64.6	195.5	38,205	2730	533700	7.45	0.00184
23	938	881 "	22,18 "	540 "	341 "	584	177	2281	357	180	1446	23.34	0.2397	64.9	139.8	19,550	3300	461300	7.12	0.00192
24	791	626 "	23.75 "	408 "	218 "	467	162	2481	304	142	1486	18.48	0.1899	65.1	110.4	12,186	3660	404100	6.83	0.00201
25															92.3	8,517	3870	357200	6.56	0.00210
26															80.1	6,416	4005	320800	6.34	0.00216
28															64.9	4,212	4080	264800	5.96	0.00230
30	327	107×10³	51.41×10⁶	90.4×10³	16.5×10³	128	99	4738	144	45		6.23	0.0640	65.7	55.2	3,045		225200	5.67	0.00242
32															48.0	2,306		195800	5.41	0.00253
34															42.5	1,804		173400	5.20	0.00264
36															37.4	1,399		152600	5.00	0.00274
38															34.5	1192		140800	4.87	0.00282
40															31.6	997.9		128900	4.74	0.00289
44															27.0	727		110200	4.51	0.00304
48	157	24.6×10³	943.7×10⁶	24.4×10³	0.21×10³	14.6	71	27980	85	14		0.90	0.009	67.0	23.5	554	4080	95880	4.31	0.00318

TABLE 8 (Concluded)

	61	62	63	64	65	66	7	8	9	0	1	2	3	4	5	6	7	8	9	0
					LB/IN.^{2}	LB/IN.^{2}														
	f		Δp/IN.			$\Sigma \Delta p$														
	0.0014 + (60)	0.01346 · (61) × 10^4	$\frac{(56) \quad (62)}{(10)}$	$\frac{\Delta p_n + \Delta p_{n+1}}{2}$	(64) · Δl															
14.0	0.00340	0.4516	7.8																	
14.5	0.00335	0.4509	7.3	7.6	3.8	3.8														
15	0.00329	0.4428	8.3	7.8	3.9	7.7														
16	0.00320	0.4307	17.2	12.8	12.8	20.5														
17	0.00311	0.4186	31.2	24.2	24.2	44.7														
18	0.00307	0.4132	43.0	37.1	37.1	81.8														
18.7041	0.00306	0.4119	45.6	44.3	31.2	113.0														
19	0.00306	0.4119	45.2	45.4	13.4	126.4														
20	0.00309	0.4159	39.0	42.1	42.1	168.5														
21	0.00316	0.4253	23.8	31.4	31.4	199.9														
22	0.00324	0.4361	9.2	16.5	16.5	216.4														
23	0.00332	0.4469	4.0	6.6	6.6	223.0														
24	0.00341	0.4590	2.3	3.2	3.2	226.2														
25	0.00350	0.4711	1.6	2.0	2.0	228.2														
26	0.00356	0.4792	1.2	1.4	1.4	229.6														
28	0.00370	0.4980	0.8	1.0	2.0	231.6														
30	0.00382	0.5142	0.6	0.7	1.4	233.0														
32	0.00393	0.5290	0.4	0.5	1.0	234.0														
34	0.00404	0.5438	0.4	0.4	0.8	234.8														
36	0.00414	0.5572	0.3	0.4	0.8	235.6														
38	0.00422	0.5680	0.2	0.2	0.4	236.0														
40	0.00429	0.5774	0.2	0.2	0.4	236.4														
44	0.00444	0.5976	0.2	0.2	0.8	237.2														
48	0.00458	0.6165	0.1	0.2	0.8	238.0														